Professional Leave Report Cover Sheet

Name: Carmen Caprau

Department: Mathematics

College: Science & Mathematics

Leave taken: ◉ Sabbatical ○ Difference in Pay ○ Professional Leave without Pay

Time Period: ◉ Fall 2021

○ Spring

○ Academic Year

○ Other

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Sabbatical Leave Report – Carmen Caprau

I was on sabbatical leave during Fall 2021. I appreciate very much the opportunity that allowed me to fully concentrate my time on research for an entire semester. Following is the report of activities I have engaged in during this sabbatical leave.

Section 1. Reporting on the Accomplishments of the Leave

Analysis of the accomplishments of the leave in relation to the goals of the original proposal.

(a) Manuscript preparations and submissions. My plan was to get to publish at least two papers (and maybe have the first draft of another one paper). I am happy to report that **I was able to finish the following three research papers which were submitted to peer reviewed journals**. Co-authors listed with an asterisk graduated from Fresno State in May 2021 with an M.S. in Mathematics.

- Jagdeep Basi*, Carmen Caprau. *Quandle coloring quivers of* $(p, 2)$*-torus links*, 14 pages; submitted to Journal of Knot Theory (under review).
  Preprint available at
  `http://arxiv.org/abs/2112.05297`
- Carmen Caprau, Bradley Scott*. *Minimal generating sets of moves for diagrams of isotopic knots and spatial trivalent graphs*, 25 pages; submitted to Journal of Knot Theory (under review).
  Preprint available at
  `http://arxiv.org/abs/2202.04839`
- Carmen Caprau, Antonia Yeung. *Algebraic structures among virtual singular braids* , 20 pages; submitted to Results in Mathematics (under review).
  Preprint available at
  `http://arxiv.org/abs/2201.09187`

I also worked on the first draft of another research paper, titled *Invariants for colored classical and singular links* co-authored with Fresno State mathematics majors, Audrey Baumheckel and Conor Righetti. By the end of December 2021, we finished the first rough draft of the manuscripts. This semester, we have continued working on this project, and we plan to finish the paper and submit it to a journal by June 2022.

(b) Presentations. I was invited to give two talks, one at the Knots in Washington in December 3-5, 2021 held virtually at George Washington University, and another one at the AMS Sectional Meeting at Charlottesville at the University of Virginia. The latter was supposed to be held in March 12-13, 2022, but it was canceled due to the rise in the Covid-19 cases between December 2021-February 2022. For the

talk at GWU, I let my collaborator, Jagdeep Basi to give the talk instead of me. I thought that this would be a great experience for Jagdeep, as he has applied to PhD programs, and it would make him a more competitive applicant. The talk was about our work on quandle coloring quivers.

- *Quandle coloring quivers of* $(p, 2)$*-torus knots links*, "Knots in Washington," George Washington University, December 4, 2021.

After Jagdeep gave his talk at GWU, he was invited to give another talk in the virtual asynchronous *Classical knots+virtual knots+algebraic structures related to knots* (CKVK*) seminar, held at The Ohio State University. His talk can be found at the link below:

`https://u.osu.edu/ckvkastrks/`

I am happy to report that Jagdeep has been accepted into the PhD program in Mathematics at The University of Iowa, and he will start the program in Fall 2022. I am very proud of Jagdeep Basi, my collaborator, and our undergraduate and graduate Fresno State math alumnus.

(c) Travel; Professional Conferences and Workshops. I started my research endeavor during the sabbatical leave with a two-week research meeting with my collaborators: Nicolle González (University of California, Los Angeles), Christine Ruey Shan Lee (University of South Alabama), Radmila Sazdanović (North Carolina State University) and Melissa Zhang (University of Georgia). We met in Raleigh at North Carolina State University between July 22–August 4, 2021. We were able to make progress on our work related to applications of the Khovanov homology to twists of braids. This is a challenging but interesting and exciting work in progress.

The sabbatical leave also allowed me to attend the following workshops and weekly seminars.

- *Foam Evaluation Workshop*, ICERM (virtual), November 5-7, 2021.
- Weekly *Online Knot Theory Seminar*; weekly presentations by experts in knot theory and its ramifications.
- *Link Homology Reading Group* (Summer and Fall 2021); readings and virtual presentations related to latest advances in link homology theories.

The original plan was to attend in person the above Foam Evaluation Workshop at The Institute for Computational and Experimental Research in Mathematics (ICERM) in Providence, RI, but due to the large number of cases of Covid-19 in the country during Fall 2021, I did not feel comfortable attending the workshop in person, and decided to participate virtually.

(d) Grant Submission. During the sabbatical leave, I **submitted the following NSF research grant proposal** on November 2, 2021. As of now, the proposal is under review.

- Carmen Caprau, *RUI: Link homology theories and other quantum invariants.* National Science Foundation in amount of $\$226,327$ (September 1, 2022 - August 31, 2025).

Over the last 20 years, the field of low-dimensional topology has been revolutionized by the discovery of powerful knot and link homology theories categorifying polynomial invariants of knots and links. The proposal aims to construct and study new Khovanov-type homology theories via webs and foams modulo relations; and investigate applications of these theories to questions about braids and cobordisms, in particular to problems related to concordance and invariants of surface-links and surface-knots. Through the proposed research, we also aim to extend known quantum link invariants to singular and virtual links, and investigate their properties and applications to low-dimensional topology

The broad range of topics in the proposed work will establish connections between various areas of mathematics, including low-dimensional topology, combinatorics, algebraic geometry, abstract algebra and representation theory. The outlined activities will foster mathematical research activities across the Mathematics Department at California State University, Fresno. If awarded, the funding provided by this grant will enable me to continue offer successful research experiences to undergraduate and Master's mathematics students, where the students are charged with investigating publication-worthy open problems, and will contribute to a departmental culture where undergraduate research is an important component of the undergraduate education. There are manageable problems for students coming from this proposal.

Section 2. Benefits to You as a Faculty Member and to the Research Community

This sabbatical leave gave me the chance to embed myself in research for an entire semester, in a way that is not possible in a regular semester. It has been very beneficial to my research program. It allowed me to work on deep problems, as well as finish projects that involved student collaborations.

My research interests lie in general in low-dimensional topology and more specifically in knot theory. Singular knots are embeddings of 4-valent graphs in 3-dimensional space considered up to rigid-vertex isotopies, and are interesting objects that enrich the field of knot theory.

In the paper with Jagdeep Basi we studied quandle coloring quivers of $(p, 2)$-torus links with respect to dihedral quandles. A quandle coloring quiver is a quiver structure which is defined on the set of quandle colorings of an oriented knot or link by a finite quandle.

The main goal of knot theory is to be able to distinguish knots and links. Reidemeister moves are local diagrammatical moves for knot and link diagrams that preserve the topology of the knot or link. When the strands of a knot or link have an assigned orientation, there are several versions of the Reidemeister moves, and thus it is ideal to work with minimal sets of such moves that generate all of the oriented versions of the Reidemeister moves. Michael Polyak proved that all oriented versions of Reidemeister moves for knot and link diagrams can be generated by a set of just

four oriented Reidemeister moves, and that no fewer than four oriented Reidemeister moves generate them all. Polyak also proved that a certain set containing two Reidemeister moves of type 1, one move of type 2, and one move of type 3 form a minimal generating set for all oriented Reidemeister moves. In the paper with Bradley Scott, titled *Minimal generating sets of moves for diagrams of isotopic knots and spatial trivalent graphs*, we expanded upon Polyak's work and provided an additional eleven minimal generating sets of oriented Reidemeister moves, and we proved that these twelve sets represent all possible minimal generating sets of oriented Reidemeister moves. We also considered the Reidemeister-type moves that relate oriented spatial trivalent graph diagrams with trivalent vertices that are sources and sinks and proved that a minimal generating set of oriented Reidemeister-type moves for spatial trivalent graph diagrams contains ten moves.

In the paper co-authored with Antonia Yeung, titled *Algebraic structures among virtual singular braids*, we proved that the virtual singular braid monoid on n strands embeds in a group VSG_n, which we call the virtual singular braid group on n strands. The group VSG_n contains a normal subgroup $VSPG_n$ of virtual singular pure braids. We showed that VSG_n is a semi-direct product of $VSPG_n$ and the symmetric group S_n. We provided a presentation for $VSPG_n$ via generators and relations. We also represented $VSPG_n$ as a semi-direct product of $n-1$ subgroups and study the structures of these subgroups. These results yield a normal form of words in the virtual singular braid group.

The results in these research papers are new and provide additional avenues for research projects.

Section 3. Benefits to the University, to our Students and Anticipated Outcomes

I believe that our Department, College and University benefit from every scholarly activity we do, whether it is a publication, a talk or a poster presentation, or a funded grant proposal. These kind of end-results and activities show that our University is committed to supporting scholarship and creative activities, as well as attracting competitive and committed educators and researchers. In addition, the professional and personal growth that is acquired by being involved in scholarly activities transfers to our students, as scholarly activity makes us better-prepared educators.

The articles that I was able to finish and submit during the sabbatical leave were in collaboration with Fresno State alumni. Not only that they increase my visibility in the knot theory research community but they also help the students who are co-authors of these papers. These students have great potential to pursue doctoral degrees in Mathematical Sciences, and having a paper published in a peer-reviewed journal will look great on their CVs/resumes and grad school or job applications.

The NSF research grant proposal, *Link homology theories and other quantum invariants*, if funded will definitely bring visibility to our University. If awarded, the funding

provided by this grant will enable me to continue to offer successful research experiences to undergraduate and Master's mathematics students, where the students are charged with investigating publication-worthy open problems, and will contribute to a departmental culture where undergraduate research is an important component of the undergraduate education.
If this NSF proposal will not be funded, I plan to use the feedback of the reviewers. and resubmit during the next cycle.

The research that I was immersed in during this sabbatical leave, and some of its results, provide new directions for research and opened new questions. Some of these questions could be tackled by talented and ambitious mathematics students, and could form the basis for future Senior Projects (MATH 198) or Master's Theses (MATH 299).

Section 4. Seminar

I gave a talk in the department seminar on Friday, March 25 at 9am, titled "On an invariant for colored, classical and singular links."

Section 5. Original Proposal

Please find enclosed a copy of my sabbatical leave proposal.