

PRIMER
OF
LOGICAL ANALYSIS:

FOR THE USE OF
COMPOSITION STUDENTS.

BY
JOSIAH ROYCE,
PH. D. (BALT), INSTRUCTOR IN THE ENGLISH LANGUAGE
IN THE UNIVERSITY OF CALIFORNIA.

SAN FRANCISCO:
A. L. BANCROFT AND COMPANY.
1881.

Phil 5057.8

Entered according to Act of Congress, in the year 1881,
By A. L. BANCROFT AND COMPANY,
In the Office of the Librarian of Congress, at Washington.

ERRATA.

The student is requested to note the following corrections, made necessary by unavoidable haste in printing:

Page 9, twenty-first line from top, insert "a part of" before "what."

Page 49, last column, third section, for " $\bar{x}\bar{y}=0$ or $=0$," read " $\bar{x}\bar{y}>0$ or $=0$."

Page 49, last line of foot-note, add to the end of the sentence the words, "outside of y or of x ."

Page 53, last column, last line of first half, for " $\bar{p} \bar{q} \bar{r} \bar{s} \bar{t}$ " read " $\bar{p} \bar{q} \bar{r} \bar{s} \bar{t}$."

Page 58, fourth line from bottom, after "terms" add, "or in being made up of new combinations of terms."

Page 64, first sentence after the first table, read, "account must be taken of the cases where $xy=0$."

Page 68, third line from bottom, add, after "A is false," the words, "but both cannot be false together."

Page 73, sixth line from top, for "Languages," read "Language."

PREFACE.

This very elementary and fragmentary little work is intended in the first instance for some of my own students in English Composition. The plan followed is suggested to me by several considerations. Every teacher of composition knows that many of the blunders of young writers are due to a defective appreciation of the meaning of language. To remedy this defect we use to some extent exercises in grammatical analysis, but expect still more from the effects of practice in writing, and of experience in general reading. All these means are valuable, in fact indispensable. But there is room for the use of other means. Why, for instance, should we not introduce, as a part of our instruction, some systematic analysis of the meaning of sentences other than grammatical analysis? Grammatical analysis is especially interested in the forms of speech, and in the traditions of the language. We might do well to study sentences purely with a view to a simple and exact expression of their whole meaning. But if we wish to do this, the best method is not far to seek. Logic, ever since Aristotle (in fact ever since Socrates), has been concerned in part with the meaning of language. Modern logical discussion, in each of its two great streams: in the German *Reform der Logik*, represented by such men as Lotze, Bergmann, Sigwart, Wundt, and in the English post-Boolean Symbolic Logic of such men as Prof. Jevons and Mr. Venn, is constantly adding something to the clearness, to the elegance, to the completeness, and so to the practical usefulness of the logical analysis of speech. Cannot some of these results be made of use for our purpose?

Composition-teachers, however, often regard Logic as too philosophic a science to be presented to elementary students. Doubtless much of the traditional logic, and yet more of modern logical discussion, is beyond the province of the composition-teacher. But may we not take up and present to our students so much of logical doctrine as bears upon the meaning of language?

Some such task I had in mind when I resolved to write these pages; and a few fragments of this task have been in this book attempted.

These lessons are therefore in no sense a text-book of Logic. Of Logic as a philosophic science they tell nothing. They treat, for instance, *Terms* as mere Names, although the author is at heart no Nominalist. They treat inference as a higher form of the interpretation of speech, wholly ignoring the philosophic problems of the theory of reasoning. But all this is a part of the plan. Concepts, viewed in their expressions alone, must be regarded as mere names, simply because we then abstract from their inner nature, and speak only of their outer expression. And just so, reasoning, expressed in speech, becomes part of the process of the interpretation of language, simply because we then regard it in no other light. The immediate aim of these lessons is therefore to form and to direct the habit of reflecting upon the meaning of speech.

Little or no genuine originality is attempted in these pages. I have tried to be independent in the choice and in the use of material; but for every idea in the lessons I am indebted to some not very remote source. Most especially must I acknowledge obligations to Sigwart's *Logik*, to Lange's *Logische Studien*, to parts of Boole's *Laws of Thought*, to Prof. Jevons' text-books, to the very suggestive essays of Prof. G. B. Halsted in the *Journal of Speculative Philosophy*, and to Mr. Venn's *Symbolic Logic*. The last-mentioned book, by its critical and lucid discussion of the whole subject, is of the very greatest aid to those who, like myself, knowing little of mathematical method, desire to get an understanding of the spirit of modern Symbolic Logic. My examples in this book are largely my own; but the labor of finding new examples is in fact now rendered almost wholly superfluous since the appearance of that great boon to all Logic teachers, Prof. Jevons' *Studies in Deductive Logic*.

BERKELEY, October 30, 1881.

TABLE OF CONTENTS.

	PAGE
CHAP. I (§§ 1-4). INTRODUCTION	7
CHAP. II (§§ 5-7). WHAT IS A STATEMENT?	18
CHAP. III (§§ 8-10). EXERCISES IN THE REDUCTION OF STATEMENTS TO THE TYPICAL FORM. CLASSIFICATION OF STATEMENTS . . .	25
CHAP. IV (§§ 11-14). TERMS AND THEIR CLASSES	38
CHAP. V (§§ 15-18). THE DETERMINATION OF THE MEAN- ING OF STATEMENTS	55

LOGICAL ANALYSIS.

CHAPTER I.

INTRODUCTION (§§ 1-4).

§ 1. THE COMPARISON OF DIFFERENT STATEMENTS AS A MEANS OF DISCOVERING THEIR FULL MEANING.

We think that we understand our own speech, but a little trial will convince us that our understanding even of very simple English sentences is often imperfect, and could be greatly improved by further study. A listener or reader is said to have a perfect understanding of any statement when he is fully aware just how much this statement adds to his previous knowledge, or, in other words, just what the statement *expresses*, and just how much it *implies*. Applying this test to our ordinary comprehension of speech, we find that though we often feel sure that we understand a sentence very well, yet we are commonly much puzzled when we have to tell how much new matter is added to our knowledge by this sentence, and whether some other sentence is equivalent to it, is implied in it, or contradicts it.

WHAT IS IT TO KNOW THE FULL MEANING OF A SENTENCE ?

To be sure, then, that we understand a sentence, we must be able to say, when any new and equally well-understood sentence is put beside the first, whether the new sentence is in meaning *equivalent* to the first, or *consistent* with the first, or *deducible* from the first, or *opposed* to the first. Yet other questions may be asked about the relative force of two sentences; but these are enough for the present. Let us explain what we mean by them.

If I say: *Bacon was not the author of "Hamlet,"* I may be required to answer the following questions:

First: Does this sentence convey the same meaning as the following sentence: *The author of "Hamlet" was not Bacon?* Or does either of these sentences express or imply more or less than the other? The answer is, in the usual way of understanding our

speech these two sentences are exactly *equivalent*; i. e., they express and imply the same meaning. You can use either you please. There may be some rhetorical reason for choosing one of the two. In sense they are identical.

Secondly: Is the first sentence equivalent to the sentence: *Shakespeare was the author of "Hamlet?"* Here the answer is negative. The second sentence does not convey the same sense as the first.

Thirdly: Is the sentence: *Bacon was not the author of "Hamlet,"* consistent with the sentence: *Shakespeare was the author of "Hamlet?"* The answer is, Yes. The two statements are not equivalent, but they are not opposed. They may be true at the same time. Statements that may be true together are called *consistent*.

Fourthly: Is the first statement consistent with the statement: *Bacon was the author of all the plays commonly called Shakespeare's plays?* The answer is, No; if by *Hamlet* we mean (as we do) one of the plays commonly called Shakespeare's. For if Bacon was the author of all these plays, then he must have been the author of that one of them called *Hamlet*. But this is denied by the first sentence. Hence the two statements are inconsistent, and one of them, at least, must be false.

Fifthly: Does our first sentence: *Bacon was not the author of "Hamlet,"* imply that some one else must have written *Hamlet*? That is, if the first sentence is true, must the statement be true that, "*Somebody, not Bacon, was the author of "Hamlet."*" The answer seems a little puzzling. Is it necessary to suppose that *Hamlet* was written at all, or by anybody? Of course we all believe that *Hamlet* had an author; but we are now asking only what our first statement, taken all by itself, can be said to imply. The answer seems to be this: The first statement, taken quite alone, does not of itself imply that anybody beside Bacon wrote *Hamlet*; but the first statement, taken with another very simple statement, viz., with the mere truism that somebody must have written *Hamlet*, does imply that: *Some other person than Bacon was the author of "Hamlet."*

Sixthly: Does the sentence: *Shakespeare was the author of "Hamlet,"* imply that Bacon was not the author of *Hamlet*? Answer, Yes; if it be understood that by the names Shakespeare

and Bacon we mean different persons, and that by the words, "*the author*," we imply that there was but *one* author of *Hamlet*. When one or more statements imply another, we say that this other is *deducible* from the first. Thus then, that: *Bacon was not the author of "Hamlet,"* is deducible under the mentioned conditions from the statement that *Shakespeare was the author of "Hamlet."*

Seventhly: Does the statement: *Bacon was not the author of "Hamlet,"* contradict the statement, *Bacon was the author of "Hamlet?"* Answer, Yes. To contradict is to deny altogether and expressly. Two contradictory statements are of course inconsistent.

The student might vary these questions indefinitely. If we understand our first statement, and the new statements that are to be compared with it, we shall always be able to answer such questions; our accuracy and skill in answering them is the measure of our understanding of our own speech. Confusion about the exact meaning of what we say often lurks concealed under our fairest speech; but such confusion appears soon enough when you begin to ask such questions as the foregoing.

In concluding this first section, we may briefly sum up what is meant by calling two statements equivalent, consistent, or contradictory, or by saying that one of the two implies the other. We shall do this by a little table.

If, then, we have taken any statement as the one with which we shall compare others, then:

A statement equivalent to the first	A statement consistent with the first	A statement implied in the first	A statement that contradicts the first
is consistent with	may or may not be equivalent to	may or may not be equivalent to	is not equivalent to
is implied in	may or may not be implied in	is consistent with	is not consistent with
is not contradictory to	is not contradictory to	is not contradictory to	is not implied in

the first statement.

§ 2. DIFFICULTY AND IMPORTANCE OF KNOWING THE EXACT MEANING OF SENTENCES.

The very simple cases that have been mentioned illustrate what we mean by consistency, contradiction, implication, and equivalence, but do not illustrate the difficulties that often arise when we try to apply these distinctions. It may seem an easy task to tell whether two statements are or are not consistent; and it would always be an easy task were we always clear about the force of our own words. But so careless are we, that to examine our own speech for the purpose of finding just what it means is one of the most difficult of exercises. No study of nature or of man can take the place of this dry work of self-examination. The lack of it produces thousands of volumes of vague, hopeless error and wrangling. The habit of being sure of what you mean is the secret of a clear style, of good thinking, of useful discussion. But the habit can be formed only through practice.

EXAMPLES OF THE DIFFICULTY.

We now come therefore to the task of exemplifying the difficulties referred to. Take the sentence: *All just men are happy*. Compare with this the sentence: *No unhappy men are unjust*; or the sentence: *No happy men are unjust*; or the sentence: *All unhappy men are unjust*. Ask yourself as to each of these sentences whether it is equivalent to the first, or deducible from the first, or contradictory to the first, or consistent with the first. Now, on reading each of these sentences, you think that you understand what it means, and just what it means. Yet on being asked to compare two of them together to see whether they mean the same thing, or two different and opposed things, or two different but consistent things, you find yourself not only hesitating, but often failing altogether. People blunder most astonishingly when they are asked such questions. Yet what is the ground for the pretense that you understand these sentences if you can not answer the question whether any two of them are or are not consistent or equivalent, or whether one says all that another says, or more, or less than another says? If one declares that he knows what a dime is, and also that he knows what a dollar is; but adds that he does not know whether or no a dime is worth more than a dollar: what shall we say of his

knowledge of our national coinage? If one knows what we mean by horse, and also what we mean by bird, must he not be able to tell us whether a horse and a bird are the same animal? And just so, if one does understand a sentence, ought he not to tell us whether it means the same as some other equally well-understood sentence? If he can not tell us this, what is his understanding of these sentences worth?

REASON WHY A FULL UNDERSTANDING OF SENTENCES IS DIFFICULT.

If we try to think of the reason of all this difficulty in answering simple questions about the meaning of sentences, we may be helped in seeing what we ought to do to conquer the difficulty. The reason why we are so imperfectly aware of what our language means is this: In speaking or in writing, in listening or in reading, we think only of such part of the meaning of sentences as for the moment is especially useful to us. If I say: *All just men are happy*, I am thinking for the moment only about just men and their good fortune; not about unjust men, nor about unhappy men; nor am I thinking whether there are in the world more happy than just men. Yet by my speech I imply something about some of these other matters. The meaning of my words extends to other cases than those that I have in mind. I am conscious of meaning some one definite thing, and half conscious, or perhaps wholly unconscious, of anything more. I have to take trouble in order to become aware that beyond what I clearly think at the moment, my words imply a good deal that I do not clearly think.

I must therefore find some way of making clear to myself my whole meaning. I must be able in some systematic fashion to analyze my speech, to see what is behind its first and most obvious sense. That I have to take trouble in order to succeed in such analysis, is no disgrace. Old habits are not to be overcome in a day; and since habit has led me to neglect reflection on the full meaning of what I say, I must correct habit by new habit.

IF WE OVERCOME THE DIFFICULTY, WHAT THEN?

But two questions may here be asked. The student may first wonder if we wish so to alter his old habits that whenever he says anything, *e. g.*, that *all just men are happy*, he shall forthwith be not only conscious of what he himself at the moment means, but also clearly aware of all that his words imply. The answer is,

not at all. We do not wish to overburden the mind of one writing or speaking. Let him think of just so much of his meaning as at the moment concerns him. But our object is to form such a habit in the student's mind that when he has written or said anything, he shall be able to discover quickly and accurately whether his words have expressed or implied any meaning that may be, in other intelligible words, brought to his attention. You are not bound to give a list of all possible things in heaven and earth, concerning which your speech may imply something. But you ought, when asked, to tell quite readily whether your speech has really implied some suggested statement about a particular mentioned thing, if you understand what this mentioned thing itself is. If you have said something and are asked, Does this that you have said imply so and so? it is a disgrace for you to pretend to understand both what you have said and what is to be compared with it, unless you are able to answer this question.

THE NEED OF OVERCOMING THE DIFFICULTY.

The student may, however, in the second place, wonder just what will be the benefit that he is to derive from such exercises. The benefit has already been indicated. If you can not become aware of what your words mean, you can not be sure of saying the same thing in any new words; you wander in obscurity whenever you write or speak; your language is apt to be confused; if the subject is difficult, you are apt to become more and more perplexed the more you work over it, and the more you multiply words. On the other hand, if you are in the habit of weighing well your words, you proceed without so much waste of effort, you know what you have said, and can say it all afresh in different words, or can avoid useless and unconscious repetitions of the same sense. And there is still another advantage to be gained from such exercises.

AMBIGUITY AND ITS AVOIDANCE.

Our language is full of tendencies to ambiguity. Words have double meaning, phrases double force, and constructions may be ambiguous in ways without number. Language, like the law of the land, is a very wise and wonderful product of centuries of progress. But we know that in practical life cases constantly arise for which old laws give no strict rule; so that knaves can

evade the law, honest men and good citizens unwittingly stumble into a violation of the law, lawyers spend months in wrangling over the law, and judges a weary time in deciding how the old law applies to these novel cases. Just so in language. Past experience has decided what shall be our general fashions of speech. These fashions the grammarians and rhetoricians try to reduce to rule. If they succeed they guide us in writing and speaking. But their guidance is imperfect. As every new business transaction may turn out to be an experiment at law, a test of the old laws, and perhaps even a ground for modifying them, so every new sentence that you write or speak is an experiment in language-building, a test of old speech-fashions to see whether they will enable you to express your new meaning. If you succeed perfectly in adapting the old fashions to your new needs, you have a good sentence. But if, either because you lack skill in applying the old fashions to your new meaning, or because the old rules of speech are inadequate, your language does not follow the old rules, the result is imperfection and perplexity. If you break grammatical rules, we may leave you to the tender mercies of the grammarians. But if, while speaking or writing grammatically, you otherwise fail to adapt your language to your meaning, the result is weakness, obscurity, or ambiguity. Of these failings the last is the most fatal, and it constantly besets inexperienced writers. How shall we learn to avoid ambiguity? The first way is clearly this: We must learn to know ambiguity when we see it. Given any ambiguous expression, we must be able to state with perfect clearness both or all of its meanings. Such practice will enable us to recognize quickly any ambiguity in our own statements, and to unravel the tangles of ambiguous speech in the statements of others. One finds, however, that students do not always readily accomplish such work. Show them an ambiguous sentence, and they are so confused by the ambiguity, which they vaguely feel, as to be unable to see clearly what two meanings the sentence has. They reverse the proverb: Because of the forest they fail to see the trees. Practice, then, in the analysis of all kinds of speech will best prepare us to overcome the difficulties of ambiguous speech. And if we carry to the work of composition the mental skill and accuracy that are to be attained through such analysis, we shall be surer of ourselves and of our ability to write well.

CLEAR WRITING AND CLEAR THINKING.

Every student of composition ought to bear constantly in mind that good writing results from clear thinking. Knowledge, practice, and a clear head produce a clear style. But the clear head is the essential thing. Without a clear understanding of what your words imply, you will write heavily and muddily; with a clear understanding of what your words imply, you will write either respectably or not at all. But constant analysis of sentences and phrases is one of the best possible means of cultivating the habit of understanding clearly the force of language.

§ 3. MEANS OF ACQUIRING THE HABIT OF ACCURATELY UNDERSTANDING SENTENCES.

The difficulties and uses of the habit of understanding what sentences mean, have been pointed out in the foregoing. We must now speak of the means by which this habit may be acquired. These means are: (1) Grammatical analysis of sentences; (2) Practice in the use of language; (3) General study of all kinds; (4) Logical analysis of sentences. To the last of these means the following lessons are to be devoted. We wish to compare in value all four.

GRAMMATICAL ANALYSIS.

Grammatical analysis is of great use in forming habits of reflection upon the meaning of sentences. Yet grammatical analysis is for the present purpose imperfect, because grammar considers less the sense than the form of language. Perfect nonsense might be perfectly grammatical. Or again, if words that have subordinate value for the meaning of the sentence chance to be put in a grammatically important place, grammatical analysis warps the sense of the sentence by making such words seem to be of the first importance. Thus if we say: *Every school-boy knows that George Washington was a great and good man*, we are doubtless not so much making assertions about school-boys, as thinking of George Washington and of his character. Yet grammatically *every school-boy* is the subject of the sentence. To bring out the true force of our words, we must therefore make some other kind of analysis than the grammatical one. Yet more, in grammatical analysis and in parsing, many matters are considered that have nothing to do with the sense of our sentences, but that have to do solely with the

traditions of our language. Nouns and pronouns, for instance, are grammatically distinct sorts of words; yet the noun *John*, and the personal pronoun *he* relating to *John*, have just the same sense, and analysis of the sense alone would not distinguish the two sorts of words.

Grammatical analysis therefore tells us both too much and too little to suit our present purpose. Yet grammar is an indispensable preliminary to any extended analysis of the meaning of sentences. Above all, Latin grammar, owing to the peculiar structure of Latin sentences, is a very fine exercise in language analysis, and must always be useful to any one that has yet to form a clear style.

THE OTHER MEANS OF ACQUIRING THIS HABIT.

Every study that cultivates clear thinking also helps to some extent to form the habit of analyzing speech, and so conduces to the formation of a clear style. But as no one ever becomes a good lawyer merely by having lawsuits of his own to prosecute or defend, so no one is apt to get a clear style merely by doing work that would be furthered by the possession of a clear style. One must reflect upon language in order to perfect himself in the art of using language. It is not enough to use speech without studying speech, any more than it is enough to be law-abiding in order to become a good lawyer. Some people, again, expect to get the habit of analyzing language merely by studying science, or by some other useful mental work. They might as well expect to become railway engineers merely by riding in railway carriages. Natural ability will do very much; but, in general, if we want to know a thing we must study that thing. Nobody becomes a blacksmith by looking at a shod horse or at a wheel-tire.

The force of our illustrations is, however, greater than has yet appeared. Language differs from the law in this, that not everybody need be a lawyer, while everybody ought to be a special student of his own speech. And language differs from the blacksmith's art in this, that we can find other people to shoe our horses or to put tires on our wheels; but can find nobody to make our own sentences. All alone and for ourselves we must frame the sentences that are to express our own thoughts; just as all alone and for ourselves we must digest our own food. Nobody can use our language for us.

Very important, simply indispensable, is of course the work of practical composition, the actual expression of our own thought. But we must not depend altogether on practice. Practice, unaccompanied by self-examination and theory, is blind. And the theory that we most need for our present purpose is: *The systematic analysis of the meaning expressed and implied in any given sentence or collection of sentences.* This theory forms a study by itself. This study we shall call *Logical Analysis*. We consider it a most important means for acquiring the habit of understanding language, and the associated habit of writing clearly. It can not supersede practice or general study; but neither ought it to be superseded by anything. It will, as said, form the subject of the following lessons. A word in conclusion as to the meaning of the name. Logical analysis must have something to do with logic. But what is logic?

§ 4. LOGIC AND ITS CONNECTION WITH LOGICAL ANALYSIS.

Let the student have patience for a moment while we lead him in a path that may seem a little new.

THINKING AND THE WAYS OR FORMS OF THINKING.

Look at the current of your own thoughts where you please, and you will find yourself running on from *assertion to assertion*. You are thinking, for example, about quail in the brush. "That is the chirp of a quail. It was in that direction. They must be in there. Not far away. If they fly up from that point, and are shot, I shall not be able to get them. The brush is too thick. No dog could get in there. Yes, my dog could. He will go anywhere. But last month he failed once to get through some such thorny underbrush as this. Besides, there are the steep rocks beyond. The quail will fall among them and never be found. I must wait till they are out." Such might be the train of thought of an amateur sportsman. What is it made of? Assertions. They follow in a stream. One of them suggests another, or gives the reason for believing another. Now this *form* of a chain of assertions or of judgments is peculiar to thinking, distinguishes thinking, for example, from mere feeling, but is independent of the subject about which you are thinking. Of quail or of quadratics, of money or of æsthetics, of stars or of steel rails, of anything in heaven or earth, you would think in the same way, viz., by mak-

ing a series of assertions leading you to some conclusion about the matter. This making of assertions is therefore *an universal fashion or form of thought*. Of such forms of thought there are a good many, and the science that picks them out, and that tells us about them, is called LOGIC.

A FORM OF THOUGHT is therefore a way in which we must think whenever we think at all. As men all have much the same sort of eyes or of hands, whatever the race or business of the men, so all have much the same sort of minds. All men walk on two legs (or else are cripples); and even so all men think in a series of assertions that are joined in some way more or less close (or else the men are of unsound mind). This that is common to all human thought, this way in which we all do our thinking, is called by a metaphor a *form* of thought.

Logic, being the science that tells us about these forms, is a very deep and extensive science. But in one of its parts this science here interests us. For human thought is expressed in language. Logic must, then, show us how language expresses the various forms of thought. And in order to do this, logic again must analyze language in order to find what the language means. And such examination of language we call LOGICAL ANALYSIS.

By means of this *logical analysis*, then, we shall find out in what way language expresses thought, and what forms of speech correspond to particular forms of thought. We shall thus be studying language as an instrument for expressing meaning, or we shall be analyzing language, *i. e.*, taking our speech to pieces, to see how and in how far it is of use to us in expressing our thoughts. This kind of analysis would be identical with that already described in the previous part of this lesson.

SUMMARY.

We finish, then, with a restatement of our two definitions, and with a summary of our previous discussion:

LOGIC is the science that tells us how men have to think, or what are the forms and fashions in which all thought must necessarily take place. Logic, in its entirety as a science, is too deep and extensive a study for our present object. These lessons are confined to the *logical analysis of speech*.

LOGICAL ANALYSIS is the process of studying the structure of sentences, in order to see precisely what meaning they express,

and how they express this meaning. Or, logical analysis is the science that teaches how language is used as an expression of thought. These two definitions come to the same thing.

Logical analysis is useful as a study of language that has been written by others, and as an introduction to composition. In fact, since in analyzing sentences you are constantly restating them in new forms, logical analysis is not only an introduction to composition, but is itself a practical exercise in composition.

CHAPTER II.

WHAT IS A STATEMENT? (§§ 5-7).

§ 5. STATEMENTS AND THEIR MANY FORMS—CONSEQUENT DIFFICULTY IN UNDERSTANDING STATEMENTS.

A statement may be for the first defined as any word, or combination of words, that is either true or false. A single word might be a statement; thus, the cry, *Fire!* is often used to express the meaning: *That light yonder is caused by a large fire.* Two words often make a statement; e. g., *fire burns.* Any number of words might be put together in one statement. The only limit is our power to understand the whole expression. But statements are always either true or false, though we cannot always be sure whether a particular statement is true or is false. Thus, if I cry, *Fire!* meaning what people generally mean by the cry, then I may be feigning, or lying, or mistaken, or correct. At all events, however, my cry does or does not express the fact, is either the truth or not the truth.

A statement as such differs from a command, a prayer, an exhortation, or a question. *Is that a fire?* is a sentence neither true nor false. *Go home,* is another sentence neither true nor false. A statement is commonly, though not always, expressed in what the grammarians call a declarative sentence, with a verb in the indicative mood.

By assertion or judgment is meant that which in the mind corresponds to the statement. The statement is the language used to convey a belief. The assertion or judgment is the belief as it is in the mind. By assertion, men do indeed sometimes mean the ex-

pression of a judgment in language; but we shall here use assertion as a name for what goes on in the mind. *I assert (or judge) that the tide is rising*: this means that I have such a belief in my mind. *I have stated that the tide is rising*, means that I have put my belief into words; have not merely thought, but said that the tide is rising.

MANIFOLD FORMS OF STATEMENTS.

Having made in my mind any assertion, I may use many forms of speech to express that assertion. The forms may be nearly equivalent in meaning. Thus, the following sentences are all of them statements of nearly the same meaning, though they differ greatly in rhetorical value:

A bad boy named John struck James.

John, a bad boy, struck James.

James was struck by a bad boy named John.

The striker of James was a bad boy named John.

John, a bad boy, was the striker of James.

James was struck by a bad boy, who was nobody but John.

The bad John, a boy, struck James.

James was the person struck by the bad boy named John.

John, who struck James, is a bad boy.

We shall soon be able to see that such sentences do differ somewhat in meaning; but the difference does not now concern us so much as the fact that any one of them might conceivably be chosen by one narrating the facts that they all express.

One of the ways whereby we may make clear to ourselves the meaning of sentences is, therefore, to express in new forms the thought that the sentences contain. And, as we see, a great number of forms are easily suggested.

But this way of making clear the meaning of sentences is very defective. The reason is that, after all, the new forms are apt to convey a shade of meaning different from the original meaning. Even in case of the same words I can often convey a different sense by altering the accent of voice with which I read the sentence.

Thus, to use an example of Bishop Whateley's, if I take Macbeth's reply to the call of the apparition:

"Apparition. Macbeth! Macbeth! Macbeth!

Macbeth. Had I three ears I'd hear thee;"

I can give this reply various meanings by altering the accent as well as by altering not the accent, but merely the tone of voice. Thus I may make the sentence mean or imply the following ideas:

If *I* had three ears (as some one else has) I would hear thee; but *I* have not three ears.

If I had *three* ears I would hear thee; but having only two ears I cannot.

Of some other things I have three, but of *ears* I have only two, and so cannot hear thee.

If I had three ears, I would *hear* thee (as it is I can but see thee).

If I had three ears, I would *hear thee*; having only two ears I shall hear somebody else.

All these various implications are possible, besides the one true meaning of the words of Macbeth, a meaning that the student may state for himself.

Much more, then, if the same sentence can mean many things, will different wordings of the same thought be in danger of meaning, or at least of suggesting, various things. If we want, then, to be quite sure of our own meaning, quite sure that we grasp the meaning of the sentences of another person, we must be ready to reduce all sentences to some simple and universal form of statement, to some form quite free from ambiguity and from confusion. Given any sentence, I must be able to tell just what it means by recasting it into a certain fixed shape, wherein there shall be no mistaking of the meanings of the parts or of the whole. Is there any such simple and typical way of making statements?

§ 6. THE TYPICAL FORM OF ALL STATEMENTS.

In algebra, when a problem is stated in words, we are expected first of all, to reduce the statement to the form of an equation or series of equations. Thus: there is a sum of money from which some one takes away 420 dollars. The remainder is shared equally among several persons, of whom each gets 96 per cent. of what he would have received as his equal share if the 420 dollars had not been first removed, before the sum was equally shared. What was the sum? A moment's inspection gives us the equation (x being the sum of money to be computed):

$$\frac{4x}{100} = 420.$$

And from this immediately follows the value of x . Now such discovery of the equation by which the problem is to be solved, what is it but the expression of the fact that we understand the meaning of the problem? An equation is the symbolic way of expressing a perfectly well-understood mathematical statement. If you know the meaning of some purely mathematical statement when it is made in common speech, the equation or other symbolical expression merely records your knowledge. If you do not know the meaning of the statement, you in vain seek for the proper equation.

Now, in ordinary language statements occur that are meant to have a precise meaning. Is there not some way, similar in uniformity and simplicity to the mathematical equation, some way of recording with perfect clearness and perfect freedom from ambiguity the meaning of any understood statement? If there is such a way, we shall be greatly helped by knowing it.

There is in fact a way, uniform, simple, and fairly exact, by which we can show to others the kernel of meaning that we think ourselves to have found even in the most complex declarative sentence. We shall have occasion after a while to compare the results of this way with the results of mathematical procedure, and we shall see the imperfections of the present way. But, for the present, the approximately exact fashion of telling the meaning of a sentence will suffice.

The way referred to is the following: Any declarative sentence, however complex; any statement or group of statements, however involved, can always be reduced to a single simple statement, or to a series of simple statements, of the form *A is B*, or of the form *A is not B*, where *A* and *B* may be anything or any notion of a thing that you please, according to the meaning of the statement.

This abstract way of speaking may seem a little puzzling. Example shall make clear our meaning.

Suppose that I say, *John is strong*. What is the meaning of this statement? Evidently this, that the person called John is a strong person. Let *A* stand for "John," *B* for "a strong person." Then the statement evidently means that *A is B*.

Suppose again that I say, *John strikes*. What is the meaning of this statement? I can still express this meaning in my typical form, if I let *A* stand for "John," and *B* for the idea, "a striker,"

main statement itself. If I give you a complicated sentence, and ask you to restate it in this strict form, you must closely attend to the main thought of the sentence before you can do what I ask; while if you do attend to and clearly grasp this main thought, you will easily perform what I ask. Thus the process of reducing complicated sentences to the typical form of statement is a very good test of your understanding of English, and a very good exercise in gaining an understanding of English. And then, secondly, in no other way can you be so sure of having stated the exact meaning of a complicated sentence. For, as we have seen, and as we know from the rhetoric text-books, arrangement of words, emphasis of voice, the associations suggested by various phrases, all these interfere to color the meaning of what we say. If, then, we recast a sentence into some familiar and pleasing rhetorical form, we shall be apt to convey by our new words not quite the same meaning as before. The danger, however, is less if we use the simple, colorless, pedantic, disagreeable form of the typical statement, *A is B*. We reduce the dangers from emphasis, from rhetorical arrangement, from association, all of them to the lowest possible degree. For, be it understood, *there is, in the typical proposition A is B, no more stress laid upon one part than upon another part of the sentence. Rhetorical rules about emphasis are to be excluded from any judgment passed on this typical statement.* *A is simply A, B simply B, is or is not,* means simply what it means. The arrangement in this typical proposition is a fixed one. *A* is the thing about which we are telling something; and of *A* we declare that it is *B*. That is all that we say. Furthermore, a third advantage of this reduction is that if I am to compare the meanings of several sentences, to see whether these sentences conflict or agree in meaning (cf. the previous lesson), I shall be at a loss how to go about my work unless I can first reduce all the sentences to some uniform and typical shape. The purpose of these lessons will therefore require some such reduction of sentences to a typical form. And, finally, a great advantage of this reduction of all statements to a typical form appears in the fact that by this means we make clearer to ourselves just what a statement is. This last advantage must form the subject of a separate section.

§ 7. THE TYPICAL FORM OF A STATEMENT AS AN ILLUSTRATION OF WHAT A STATEMENT IS.

A statement, by § 5, is a word or collection of words that must be either true or false. By § 6 we find that statements are reducible to the typical form *A is or is not B*. A and B are any words, or groups of words, with one condition only, viz., that the word or group of words shall be a noun, or a substantive phrase, or a substantive clause. A and B are then *names*. A statement is therefore a grouping of two names with an *is* or an *is not* between them. A statement tells us that the objects or ideas called by the first name are also called (or not called) by the second name. *John strikes*, i. e., *John is a striker*, or the person called by the name John is also called by the name *striker*. *'Tis better to have loved and lost than never to have loved at all*, means: *To have loved and lost is a thing better than never to have loved at all*; and this means that the state or condition that is called by the name before the *is*, ought, according to the poet, to be called by the name after the *is*. And so we might go on indefinitely. But what is our result? This: Any sentence in which there is one statement must be such that we can divide it into parts, each of which has the force of a noun; and every sentence that contains one statement must be such that we can express the whole meaning of the sentence by uniting these two nouns by an *is* or *is not*. Or, again, *a statement always tells us that two names have or have not, in whole or in part, the same application*.

SUGGESTION OF A TEST OF CLEAR WRITING.

These things being so, if a sentence pretends to contain one statement, but if at the same time we cannot divide the sentence into two substantive parts, connected by *is* or *is not*, the sentence is faulty, and must be corrected. Errors of this sort are very elementary; but they do occur in the essays of careless and inexperienced persons. Thus, a sentence that lacks a grammatical predicate, expressed or understood, is in the above respect lacking in sense. Less childish, however, though still elementary, is such a mistake as sometimes occurs in the use of the passive voice. *Yonder peak has often been referred to, but has not yet been told the name of*. This very glaring example illustrates the mentioned blunder. Everybody sees the blunder; not everybody

readily explains such a matter. Usage, of course, would be decisive on such a point; but, usage apart, let us examine the sense of the sentence. This is a sentence and must be reducible to the form *A is B*. *A* is "yonder peak." *B* is in the first half of the sentence, "an object that has been referred to;" in the second half, *B* is "an object not yet told the name of." This second name given to the thing that is called "yonder peak," is in reality no name at all. *Object* is a substantive, expressing a clear idea. The idea *seen object* is clear. *Object told of* is also a clear idea. *Object told the name of* expresses no idea, but a confused jumble of ideas. *Object told of in respect of its name* would be an awkwardly expressed, but clear idea. *Object whose name has been told* would also be a clear idea, and one better expressed. The confusion of the original statement would be possible only to one that had never reflected on the nature of a statement.

But our purpose just now is rather to give the student a certain habit of mind than to teach him at this point any particular rules of composition. If we make clearly conscious what has been for many of us an unconscious process, namely the process of discovering when and why a given statement is complete and sensible, the result cannot but be of the utmost importance for the art of composition.

CHAPTER III.

EXERCISES IN THE REDUCTION OF STATEMENTS TO THE TYPICAL FORM. CLASSIFICATION OF STATEMENTS. (§§ 8-10.)

§ 8. GENERAL CONSIDERATIONS BEARING UPON THE REDUCTION OF STATEMENTS TO THE TYPICAL FORM.

a. DOUBLE MEANING OF THE VERB "TO BE."

The verb *to be*, as has long been observed by logicians, has two meanings: *To exist* is the first meaning. But in our typical statement *A is B*, the *is* does not imply that either *A* or *B* has any existence. The sentence, *round squares are impossible*, exemplifies this use of *to be* as the expression of the simple act of affirming something. The two uses of the verb *to be* appear in the same sentence in the familiar quotation: *Whatever is, is right*.

The first *is* in this sentence expresses existence. The second *is* of the sentence connects the predicate adjective with the subject. In the typical statement, *A is B*, *is* has the second meaning only.

As a consequence, whenever a statement occurs that contains *is* as a verb of existence, this statement needs some alteration to reduce it to our typical form. *The republic still is*, means, therefore, when put into the typical form: *The republic is a still existent thing*. Here the *is* no longer expresses existence, and the notion of existence has been put into the part of the sentence succeeding the *is*.

b. SUBJECT, COPULA AND PREDICATE.

When we have reduced any statement to the typical form *A is B*, *A* is called the *subject*, *B* the *predicate* of the statement; and the *is* is called the *copula*.

c. THE COPULA ALWAYS IN THE PRESENT TENSE.

The student may ask what parts of the verb *to be* may serve as copula. The answer is: All the persons of the present tense, singular and plural numbers. The difference of *am*, *is*, and *are*, is a difference, not in sense, but in form: a grammatical, not a logical distinction. If the language were to drop this distinction, and were to permit the form *is* in all three persons and in both numbers, no logical obscurity would result; the sense of our statements would be quite clear and wholly unaltered.

But the other forms of the verb *to be* cannot be regarded as having the same force as the present tense, and are not pure copulas. If I say *A was B*, the *was* combines two offices, the office of the copula *is*, and the office of a temporal adverb of past time. The sense of the sentence may therefore be expressed in the form, *A is a thing having been B*. As the secret societies speak of a "Past Grand Master," or as an Englishman might speak of a man as "sometime Fellow of Christ Church College," or as we all speak of the "late lamented So-and-So," just so we may say: *A is a past or quondam B*. Such would be the meaning of the statement *A was B*. In like fashion the statement *A will be B* is reducible to the form *A is a future B*.

d. GENERAL RULE FOR REDUCING TO THE TYPICAL FORM.

The actual work of reducing any statement to the typical form is one that depends altogether on our understanding of the

statement. But to aid the student in gaining such an understanding, certain general suggestions may be useful. The most general of these is the following rule: Notice in the given statement what is the thing concerning which we are to receive the *principal new information* that the statement contains. The name of this thing, including all the modifying words that help to form or to limit our idea of the thing, will be the subject A. Notice, in like fashion, all the parts of the statement that go to form or to modify the idea of that which the subject is asserted to be or not to be; these in proper combination will form the predicate of the statement. Combine the subject and predicate by *is* or *is not*, *are* or *are not*, *am* or *am not*, as the sense may require. The result will be the statement reduced to the typical form. One ought to add, as a general caution, that the grammatical subject must not be mistaken for the logical subject, and that we must consequently look for the words that name the thing concerning which the statement undertakes to tell us something, not for the words that are in the nominative case.

e. OF THE LOGICAL CLASSIFICATION OF SENTENCES.

For the purpose of reducing complicated statements to the simple form, a preliminary classification of sentences will be useful. This classification will consider sentences as containing statements, and will group the sentences according to the various ways in which they accomplish their work as vehicles for the expression of assertions. We exclude, therefore, interrogative and imperative sentences, except in so far forth as they may be rhetorical artifices for the expression of assertions.

Sentences may contain one statement or several distinct statements. A sentence containing one statement we shall call a *Logically Simple sentence*. Grammatically, such a sentence may be complex or simple. *Lazarus died*; *Blessed are the merciful*; *It is an ill wind that blows nobody good*; *Thus conscience doth make cowards of us all*, are examples.

Slowly comes a hungry people, as a lion, creeping nigher,
Glares at one that nods and winks behind a slowly dying fire,

is a sentence containing but one statement.

A sentence that contains several distinct statements we call a *Logically Composite sentence*. *There was a man in the land of*

Uz, whose (=and his) name was Job, is an example. Another example is :

Love took up the harp of Life, and smote on all its chords with might;
Smote the chord of Self, that, trembling, passed in music out of sight.

The lines A and B are respectively equal to their opposites C and D ; this is a composite sentence, containing two statements, expressible by the equations, $A=C$, and $B=D$.

Sentences are again divisible in another way into conditional and unconditional sentences, according as they represent some statement or statements as true under certain conditions, or as true without mentioned condition. Unconditional sentences are called Categorical. Our typical form of statement is itself an unconditional or categorical sentence. Conditional sentences are either expressly conditional or by implication conditional. Expressly conditional sentences are called Hypothetical. *If 'twere done when 'tis done, then 'twere well it were done quickly*, may serve as a well-known though difficult example of an hypothetical sentence. *If A is B, C is D*, is a general expression of the hypothetical form. The condition may be expressed as a temporal condition, *i. e.*, as one at times present, *e. g.*:

My heart leaps up when I behold
A rainbow in the sky.

Or again : *While the sun shines, the birds sing.*

Of sentences not expressly, but by implication conditional, the most important examples are the so-called Disjunctive sentences. These are sentences that affirm one of two or more statements to be true, but that do not tell us which is the true one, or that leave us to suppose that sometimes one, sometimes another of the statements is true. *Either Mohammed is a false prophet, or the Koran is the word of God*, would serve as an example. Other examples are : *Animate life began on this planet either by a miracle, or by means of a natural evolution from inanimate nature ; This murderer is either insane or worthy of death ; This student is always either reading or reflecting.* How these sentences are implicitly conditional we shall see better hereafter.

Sentences are again classified as affirmative or negative. This classification, one of very great importance, is sufficiently illustrated by the two shapes given to our typical form : *A is B, A is not B.* We shall say much of it hereafter.

Sentences are once more classified according as they (1) simply make statements, or (2) add to the statement some such idea as that it is necessarily or probably true. *Cæsar paused on the banks of the Rubicon*, simply declares something as a fact of history. *This straight line A B is necessarily the shortest distance between A and B*, adds to the mere assertion the thought that this assertion is a necessary truth. Sentences of this latter class are called Modal sentences.

We repeat in a summary the foregoing methods of classifying sentences. Sentences are divided in the following four ways :

- | | | |
|--------------------|---|---|
| I. Sentences are | { | Logically Simple. |
| | { | Logically Composite. |
| II. Sentences are | { | Unconditional. |
| | { | Conditional. { Expressly: Hypothetical. |
| | | { Implicitly. { Disjunctive. |
| | | { Other Forms. |
| III. Sentences are | { | Affirmative. |
| | { | Negative. |
| IV. Sentences are | { | Merely Declarative. |
| | { | Declarative with the added Idea of Necessity, or of Probabil-
ity, or of some like quality: Modal sentences. |

These four ways of dividing sentences are actually not wholly independent; but we shall see their relations more clearly hereafter.

§ 9. SPECIAL SUGGESTIONS ABOUT THE MEANING OF SENTENCES : SIMPLE SENTENCES.

a. NON-CONDITIONAL SIMPLE SENTENCES AND THEIR CLASSES.

When we have to reduce to the typical form non-conditional simple sentences, there may sometimes arise a doubt whether a given sentence is best interpreted as having an affirmative, or as having a negative force. The rhetorical use of the double negative in the following sentence will exemplify this perplexity: *This man cannot fail to attain his end*, may be reduced thus: *This man is certain to attain, etc.*, or thus: *This man is not otherwise than certain to attain, etc.* In such cases we must decide from the context whether the primary and real intent of the sentence is to affirm through the mere artifice of a double denial, or whether the primary intent is to deny something that itself happens to be expressed negatively. If I say, in response to an invi-

tation, *I shall not fail to go*, my real purpose is probably to affirm, with the emphasis of a double negation, my intention to go. But if I am speaking with a friend about the conduct of some one whose acts seem to my friend unkind, and if I say to my friend: *No, you are mistaken; these acts are in truth not unkind*: then my sentence is primarily intended as a denial of my friend's belief, and only in the second place do my words affirm the kindness of the acts that are under discussion.

In interpreting sentences for the purpose of reducing them to the typical form, it is of importance to notice that logically simple statements may have the following purposes: (1.) Such statements may be sentences that name something, or that express a simple recognition: *This is the place. The Emperor of Russia is (or is called) the Czar. Yonder is San Francisco.* (2.) They may be sentences that declare something to possess a certain quality: *Thou art so full of misery. The stars were bright.* (3.) They may be sentences that describe an action. Such are the following lines, if each one of the lines is for the moment viewed as forming a simple sentence by itself:

The poet arose,
He passed by the town and out of the street;
A light wind blew from the gates of the sun,
And waves of shadow went over the wheat.

(4.) Simple unconditional statements may tell us of a relation between two or more objects, *e. g.*: *The dew is on the grass. A is equal to B. I have not been thy dupe, nor am thy prey. Above the forest rises the snowy mountain. This is the Queen, not of Russia, but of England.* All these sentences aim to tell us that some object stands, or does not stand, in a particular relation to some other. The relation of the dew to the grass, the relation of A and B, the relation of the speaker to the person addressed, the relation of the mountain to the forest, the relation of this Queen to England. Such are the matters concerning which information is in these statements given. (5.) Simple unconditional statements may tell us of a *state* of some object: *e. g.*, *He is awake.* (6.) Simple unconditional statements may separate the qualities or actions or relations of objects from the objects themselves, and tell us something about these qualities, actions, or relations: *Honesty is the best policy. Alas for the rarity of Christian charity*

under the sun! (Here an exclamation amounts to a statement.)
Sweet are the uses of adversity.

Sweet love, that seems not made to fade away,
 Sweet death, that seems to make us lifeless clay;
 I know not which is sweeter, no not I.

Such statements are called abstract statements. (7.) All the sub-classes thus far mentioned tell of some fact that exists at a certain time, but that may not exist at another time or under other circumstances. The seventh sub-class of simple non-conditional statements includes those that explain or illustrate the meaning of a word, and that are therefore independent of time and of place, *e. g.*: *Charity suffereth long and is kind. A triangle is a plane figure having three sides. Man is rational.* Another way of dividing simple non-conditional statements there indeed is, but owing to certain difficulties regarding the distinction of certain forms of simple from certain of the logically composite statements, we for the moment pass over this division (*cf. infra*, § 9, *d*). To summarize our division thus far, we have mentioned of the logically simple statements:

- | | |
|----------------------------|----------------------------|
| 1. Statements of Name. | 4. Statements of Relation. |
| 2. Statements of Quality. | 5. Statements of State. |
| 3. Statements of Action. | 6. Abstract Statements. |
| 7. Explanatory Statements. | |

To reduce these to the typical form, we note in each case the class to which the statement belongs, separate the name of the subject and prefix this name to the copula, and place after the copula the name of the quality, action, etc., stated in some form equivalent to the forms used in the original sentence.

b. LOGICALLY SIMPLE CONDITIONAL SENTENCES.

The difficulty of logically analyzing conditional sentences is usually for beginners considerable. Yet the whole difficulty can be reduced to somewhat insignificant proportions by a few reflections. A sentence expressly conditional begins with an *if*-or *when*-clause, and adds to this the statement of the consequences of the truth of this conditional clause. *If I drink this poison I shall soon die. When the north wind blows the roses wither.* In such sentences as these something is affirmed or stated. What is this something? In the first example just given, I do not affirm

that I shall soon die, and I do not affirm that I shall drink the poison. Both statements may be false. But the whole statement is true. What, then, do I state? I state that the consequence of drinking this poison is death. In the second example I do not affirm or deny that the north wind is blowing, nor do I affirm or deny that the roses are withering. I affirm that the consequence of the blowing of a norther is the withering of the roses. Thus, in both the examples, my true subject is the consequence of something; my predicate is the statement of the nature of this consequence. Simple statements that are expressly conditional, are therefore, reducible to the typical form as soon as we notice that they mean that *the consequence of M is N*, where M and N are any appropriate names, or name-phrases or clauses, the one naming the condition, the other the consequent.

That this interpretation is fair, we may easily see by any example of an expressly conditional sentence. But in some cases, a still simpler method of reduction to the typical form is possible. Thus in the sentence: *If a man gambles he is not to be trusted*, the assertion made is equivalent to the assertion expressed by the statement: *A gambler is not to be trusted*. Again the statement: *If that apple is sour I do not want to eat it*, is reducible to the form: *A sour apple is not an apple that I like to eat*. And, in general, the statement: *If M is N, M is also P*, can be reduced to the form: *Any M of the N-kind is P*, where N is an adjective-modifier of M, and where the whole expression is reduced to the typical form.

In sum, then, an expressly conditional sentence is reducible to the typical non-conditional form as soon as we reflect on the meaning of the one assertion that is implied in even the most elaborate conditional statement.

The Disjunctive statements are more puzzling, and in fact not simple statements. *The visible heavenly bodies either are self-luminous, or are comparatively near to self-luminous bodies, or both at once*. This statement, we have said, is indirectly conditional. At all events, this statement is not in the form *A is B*, and seems to resist reduction to that form. But let us look closer. If a visible heavenly body is not self-luminous, what do we know of it? We know from the above statement that such a body is comparatively near to some self-luminous body. The

above statement, then, might be restated thus, reducing the expressly conditional statement now reached to the typical form: *Visible and not self-luminous heavenly bodies are comparatively near some self-luminous body.* But this new statement is not a perfect expression of the whole meaning of the previous statement. Disjunctive sentences resist reduction to the typical form simply because they are composite sentences. Containing two or more statements they cannot be reduced to one.

c. OF MODAL STATEMENTS.

When I say: *This is the right road*, my statement is a simple or unmodified expression of my belief. When I say: *This must be the right road. This is probably the right road. This may be the right road. This cannot possibly be the right road*: in all such cases I give my statement, what is technically called Modality: that is, I add to the copula of the statement some adverb indicating the degree or the fixity of my belief. Such statements offer special difficulties. To reduce to the form *A is B*, by leaving out the modal adverb *probably, possibly, necessarily*, etc., is to alter the meaning of the actually made statement. Yet the adverbs *probably*, etc., do not belong as modifiers to the predicate. What kind of road would be a *probably right road*? Evidently a road is either right or wrong, and *probably* modifies not the rightness of the road, but my expression of belief. Hence, to escape the dilemma, the best way would seem to be to regard the modal adverb as a whole predicate by itself; the subject being a clause giving the statement in its non-modal, merely declarative form: *That this road is the right one, is probable.* In like manner the statement: *This road must be the right one*, would be reduced to: *That this road is the right one, is necessary.*

d. THE TRANSITION FORMS BETWEEN THE SIMPLE AND COMPOSITE SENTENCES.

Often it is a mere matter of punctuation, and one not capable of being reduced to rule, whether a given series of clauses shall be regarded as separate sentences, or shall be regarded as making up one composite sentence. We need not dispute over such cases; for logical analysis is little concerned with the mere freaks of composition. But there are sentences that seem to lie on

the boundary between simple and composite sentences, and that are of a nature to require special attention. Such are sentences with plural subjects. If I say: *You laughed, he laughed, and I laughed*, the expression is, according to punctuation, a series of simple statements or a single composite sentence, whichever you will. Suppose it to be the latter. Now the sense of this composite sentence, the force of all three of its distinct statements, could be expressed in the much briefer form: *We all laughed* (supposing only that the *we* is understood by the reader to mean the *you, he, and I*, and no other persons). Is this new statement simple? It is certainly equivalent to several distinct statements taken together. But is it composite? It is certainly a sentence with one subject and one predicate.

The number and importance of such statements the student will at once recognize when he thinks of the vast number of sentences with plural verbs. In all such sentences we have cases where a single clause is doing the work of many statements, by summarizing in brief what it would take all the many statements to express at length. If I say: *Seventeen arrests were made by the police*, my statement is false unless it sums up just seventeen simple statements of the form: *This arrest was made, this, this, etc.* If I say: *All men are mortal*, or, *All planets revolve in elliptic orbits*, my statement is false, unless every one of a great number of simple statements is true. These simple statements are summarized in the general one. They would be of the form: *This man is mortal. This planet revolves in an elliptic orbit.*

Nevertheless grammatical and yet other considerations triumph, and such plural statements are regarded not as composite, but as simple. The many subjects are gathered up in our minds into one, and of this one we affirm or deny. The resulting statement is the equivalent of several statements, but is not explicitly composite.

The mention of these composite-simple sentences enables us to complete our list of the classes of logically simple sentences, as these were mentioned in the first part of this section (v. § 9, *a*). The simple sentences there considered all had for their subjects single objects or qualities. But many objects or qualities may at the same time be made the composite subject of which something

is told; and, as a result, we have three forms of simple sentences when these are classified according to the simple or composite character of their subjects.

Logically Simple Statements : { SINGULAR: A is B; A is not B.
PARTICULAR: Some A's are B's.
UNIVERSAL: All A's are B's.

In any one of these three classes might be found statements of any one of the classes mentioned in § 9, *a*.

§ 10. SPECIAL SUGGESTIONS REGARDING THE MEANING OF COMPOSITE SENTENCES.

A composite sentence ought to be reduced to a series of co-ordinate statements, *A is B, and C is D, and E is F*, etc. The student will have already noticed that subordinate clauses do not make a sentence logically composite, and that a sentence grammatically as complex as you will, having for example twenty subordinate clauses, might still be logically simple, because it might contain but one statement. To be composite, a sentence must contain several distinct statements, and hence, must be reducible to a series of co-ordinate statements of the typical form.

The following are some of the ways in which a sentence may be composite:

a. By stating at once a general fact and exceptions to this general fact. Example: All excepting John were present. Reduced form: *John is a person not having been present; all the others are persons having been present*: two distinct statements.

b. By affirming or denying something of a certain subject while at the same time confining the affirmation or negation to this subject, and expressly excluding other subjects. Example: *None but the brave deserve the fair*. Reduction: *The brave are deserving of the fair. No persons not brave are deserving of the fair*: two distinct statements.

c. By applying the same predicate to several expressly distinct subjects. Example: *A cargo of coal, a good ship, and three men were lost in the storm*. The reduction is obvious.

d. By applying to the same subject several distinct predicates. Examples: *Our birth is but a sleep and a forgetting. And Job took up his parable and said*. The reduction is obvious.

e. By uniting several co-ordinate clauses, with or without con-

nectives. *The way was long, the wind was cold, the minstrel was infirm and old.* Any grammatically compound sentence is an example. Reduce each member separately.

Thus far we have mentioned the classes in the order of their resemblance to simple sentences, beginning with the forms most like the simple sentence in external appearance. There remain, besides the disjunctive form mentioned in the last section (this form will be better understood hereafter), two disguised forms of composite sentences that have not found a place in our list :

f. Co-ordinate clauses are sometimes introduced in the form of relative clauses. Example: *I met my brother, who (=and he) had just seen you.*

g. Co-ordinate clauses are sometimes disguised by apposition or by participial or similar constructions. *Beloved by all, the king died. Tired of all these, for restful death I cry* (Ought this to be interpreted as a simple sentence?).

In both these cases the reduction is obvious if the meaning is understood.

EXAMPLES FOR PRACTICE.

[The following sentences are to be reduced to the typical form. If they are composite, the student should point out in what one (if any) of the above-mentioned ways they are composite. If they are simple, they should be classified as in § 9. In either case, the student should say whether the sentence is or is not conditional, affirmative, or modal. After a few exercises, the student need not undertake to reduce any farther such sentences as contain merely a subject and a neuter verb, or such as connect subject and predicate by a past or future tense of *to be*; since these cases are sufficiently near the typical form for most purposes.]

1. These two—they dwelt with eye on eye,
 Their hearts of old have beat in tune,
 Their meetings made December June,
 Their every parting was to die.
2. There is nothing either good or bad, but thinking makes it so.
3. Truth loves open dealing.
4. Had I but served my God with half the zeal
 I served my king, he would not in my age
 Have left me naked to mine enemies.
5. It is impossible, within my limits, to discuss the question fully.
6. Dryden and Pope are regarded by most as poetical classics.
7. Smalē fowlēs maken melodie.
8. A knight there was, and that a worthy man.
9. Griseld is dead, and eke her patience,
 And both atonēs buried in Itaille.
10. Darkness was upon the face of the deep.
11. For thy sweet love remembered such wealth brings
 That then I scorn to change my state with kings.

12. And other strains of woe, which now seem woe,
Compared with loss of thee will not seem so.
13. Some may be near unto goodness who are conceived far from it; and many things happen, not likely to ensue from any promises of antecedencies.
14. I hold that the devil doth really possess some men, the spirit of melancholy others, the spirit of delusion others.
15. Again, I believe that all that use sorceries, incantations, and spells, are not witches.
16. Let me make the songs of a people, and I care not who makes their laws.
17. They who to states and governors of the commonwealth direct their speech, I suppose them not a little altered and moved inwardly in their minds.
18. You see her eyes are open.
19. In behint yon auld fail dyke,
I wot there lies a new-slain knight;
And naebody kens that he lies there,
But his hawk, his hound, and lady fair.
20. I dare do all that may become a man; who dares do more is none.
21. That is Laertes; a very noble youth.
22. If it be now, 'tis not to come; if it be not to come, it will be now.
23. She swoons to see them bleed.
24. 'Tis a notorious villain.
25. It is the bloody business that informs thus to my eyes.
26. Then must you speak
Of one that loved not wisely, but too well;
Of one not easily jealous, but, being wrought,
Perplexed in the extreme; of one whose hand,
Like the base Judean, threw a pearl away,
Richer than all his tribe.

27. In the cool moments of reflection, Julian preferred the useful and benevolent virtues of Antoninus; but his ambitious spirit was inflamed by the glory of Alexander; and he solicited, with equal ardor, the esteem of the wise, and the applause of the multitude.

NOTES ON THE FOREGOING EXAMPLES.

On 1: Notice that the real subject of the lines is the two lovers; and that the change of subject to *hearts, meetings*, etc., is a change for poetic effect. All the statements have the same logical subject, and this should appear in the reduction.

On 2: Notice the force of the double negative: *nothing . . . but*.

On 4: Consider carefully what is the principal subject. There are more ways than one of stating the meaning in a non-conditional form.

On 6: Point out the difference of meaning produced by putting the emphasis first on *Dryden and Pope*, then on *most*.

On 8: Remember the double meaning of *to be*.

On 13: *Conceived* = *supposed to be*.

On 14: Is *I* the logical subject?

On 15: The same question. *All are not* = *not all are* = *some are not*

On 16: *Let me make* = *if I make*. Same question about *I* as in 14.

On 18: A similar question: Is *you* the logical subject?

On 21: Composite statement; use of apposition.

On 22 and 23: Consider the real meaning of the infinitives.

On 24 and 25: What is the force of *it* in each of these sentences?

CHAPTER IV.

TERMS AND THEIR CLASSES. (§§ 11-14.)

§ 11. NATURE OF TERMS : CLASSIFICATION OF TERMS.

In the typical form *A is B*, *A* and *B* are called the *terms* (*i. e.*, *end-words*) of the statement. Logical analysis must take account of the nature of these terms themselves. The branch of logical analysis that treats of terms may be called **TERMINOLOGY**.

WHAT EXPRESSIONS MAY BE TERMS.

A term, as we have seen, is a name. Grammatically, a term may be a word, a phrase, or a clause. On the other hand, however, not every word, or phrase, or clause is or can be under all circumstances a term. Prepositions, for example, are not as prepositions capable of being either subject or predicate in a statement. The same is true of conjunctions. But adjectives, pronouns, certain adverbs, and of course nouns, can be full terms all by themselves. Words that need other words with them in order to make up whole terms, we may call **Partial Term-Words**. Words that can stand alone as terms, we may call **Whole Term-Words**. *Dog*, *good*, *up*, may be **Whole Term-Words**. *Of*, *to*, *and*, are in nature **Partial Term-Words**.

CLASSIFICATION OF TERMS.

Terms may be names of *Things*, of *Qualities*, of *Actions*, of *Relations*, or of *States*. A *Thing* is a group of facts so separate from other facts, or so permanent amid the changes of other facts, that we are inclined in our thoughts to grant this group of facts an independent existence, such as we ourselves have. A stone is a thing, and so is a cloud. One is less inclined to call a wave a thing; still less inclined to call a breath or a flash of lightning a thing, and not at all apt to call a throb of pain or a twitch of a muscle a thing. Complexity, separateness, and permanence are the chief characteristics that together lead us to regard a group of facts as a thing for itself. Of these characteristics, at least two together seem necessary to the idea of a thing. That which many different things have in common, and which at the same time endures a good while with little change, we call a *Quality*. Lumps of sugar

are things; their sweetness, a common characteristic that endures in each lump, is called a quality. Men are things, their mortality is a quality. A quality cannot exist alone by itself; a thing can exist or be thought of by itself. Qualities, however, are always found in things; and things cannot exist without qualities.

Along with the more permanent qualities in things, there go other facts that are more transient. A lump of sugar is sweet as long as it is a lump of sugar; but it may be now warm and now cold, now falling, now lying still, now my property, now yours, now in Oakland, now in Berkeley: and yet all the time it is none the less a lump of sugar, and sweet. The transient facts that may thus be continually altered without destroying the things, and without changing their qualities, are called, according to circumstances, *states*, *actions*, or *relations*. The differences among the three may be best understood by examples taken from our own personal experience. I am a thing: *i. e.*, I am complex, separate from other things, enduring. I have certain qualities: *e. g.*, I am mortal, two-legged, of American birth. These facts are comparatively permanent. But certain facts in my case are not permanent. Sometimes I am sleepy, sometimes warm, sometimes pleased, sometimes wet, sometimes ill. All these transient facts are my *states*. Other transient facts there are in my case. Sometimes I raise my arm, sometimes eat, sometimes walk. Transient facts of this sort are called my *actions*. Again, there are other transient facts, such as that I am near a table, above the sea-level by so many feet, stronger or weaker than some fellow-being, and thus related to other things. These facts are called my *relations* to other things.

INDIVIDUAL AND GENERAL TERMS.

Terms that name any of these objects may name one object, or any one of a group of objects. *Socrates* names a single thing. *Philosopher* names any one (we know not what one) of a group of things. Terms that name a single object are called *Singular terms*, *Individual terms*, or *Proper names*. Terms that name any one of a group of objects are called *General terms*, *Class terms*, or *Common names*.

When a term names any one of a group of objects, whether these objects are things, qualities, states, relations, or actions, the objects are always grouped together in our minds, *on account of*

their similarity. The name stands for any object that has certain properties; and these properties are had in common by the whole group of objects for which the name stands. *Man, color, life, possession, flight,* are such general terms. *Man* is a general *thing-name*, and names any one of a certain group of things. These objects are grouped in our minds because of their likeness in certain respects: *e. g.*, because of their rationality or their two-leggedness. To call an object a man is to say that he has these properties. *Color* is a general *quality-name*, and names any one of the qualities, *redness, greenness, blueness,* etc. These objects are grouped in our thought by reason of their likeness. *Life* is a general *state-name*, and names any one of a large group of states, including, for example, *animal life* and *plant life*. *Possession* is a general *relation-name*. *Flight* is a general *action-name*, naming equally the flight of birds and the flight of escaped convicts. All the objects for which any one of these general names stands are grouped in our minds into a class, and that by reason of their similarity.

§ 12. EXTENSION AND INTENSION—DEFINITION.

Any general term, as *man*, applies, we see, to many objects, and applies to them because they are in many respects alike. A general term, therefore, reminds us of the objects, any one of which this term is to name, and also of the properties that these objects have in common. There are the objects called men, and the properties possessed in common by men, and called human properties. Now, the collection of the objects that are named by a term is called the *Extension* of this term. The group of properties possessed by the objects that the term names is called the *Intension* of the term. The Intension is the measure of *how much the term means*; the Extension is the measure of *how many things the term names*. *Frenchman* means more than *man*, since a Frenchman must have all the common properties of humanity, and must add the French properties as well. But *Frenchman* names fewer beings than *man* names. Frenchman has therefore more Intension and less Extension than man. The Intension of a term is sometimes called its *Connotation*; the Extension is sometimes called *Denotation*.

DEFINITIONS.

To define a general term is to tell what it means, *i. e.*, how it is used. We know what a term means when we know what it *con-*

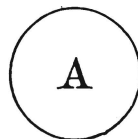
notes, i. e., what its intension is, even though we are ignorant of a great part of its extension. I have not seen all men, much less all logarithms or triangles. Yet I know fairly well what is meant by the term *man*, and I know quite exactly what is meant by logarithm and by triangle. For I know the *intension* of these terms. To define a term I must, then, tell what are the common properties possessed by the objects that this term names. A definition is a sort of declaration of our purposes as to the use of a certain word. We say, in a definition, "The name *N* shall hereafter be applied to anything that is found to have the properties *p, q, r*; and this name shall be applied to nothing that has not the properties *p, q, r*." "The name *plane triangle* shall be applied to any object that is a plane figure, and that has three sides, and to no other object."

Into the subject of the best methods of definition we can not here enter. Our brief sketch of the nature of terms must pass very lightly over many important points.

When we have once defined a term, we have generally the means of picking out from the world as many as we please of the numerous objects that the term names. Thus, having defined what I mean by triangle, I can notice as often as I please that this or that figure is a triangle, and can thus discover a part of the extension of the term triangle. The sum total of all the objects that are named by the term, the whole extension of the term, will be the class or group of objects corresponding to the name.

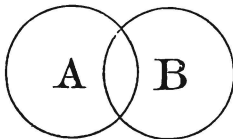
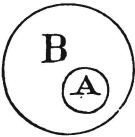
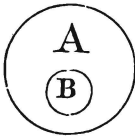
§ 13. THE RELATIONS OF TWO TERMS.

Two different names will, in general, be found to have different connotations or intensions; and hence, in general, different classes of objects will be denoted by these terms; *i. e.*, the extension of the terms will not be the same. Let the two terms be *A* and *B*. Then, in general, the names *A* and *B* will not have the same extension nor the same intension; but will name different classes of things. Now we may, by a very little stretch of the imagination, suppose either of the two classes to be represented by a circular figure, such as is seen in the adjoining diagram. That is, we may suppose that all possible objects in the universe, that are according to the definition of *A*, to bear the name *A*, are collected inside the circumference of this circle; and that no other



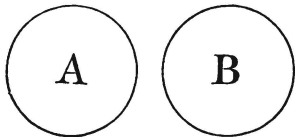
objects are inside this circle. If A means *man*, then are all possible men inside of A, and no other beings are there. If A means *color*, then all the color-qualities, and no other objects, are inside of A; and so on. If B is represented in like fashion by a circle, then the possible relations in space of the circles A and B will in some degree correspond to the relations of the classes of objects that are respectively called A and B. In particular, five possible relations of the classes, and the five corresponding relations of the circles, need our attention.

(1) The classes A and B may be wholly identical; *i. e.*, whatever object is called A may also be called B, and whatever is called B may also be called A. Such classes are *Man* and *Rational Animal*; or again, *Plane Triangle* and *Plane Figure having three sides*. The two classes, A and B, form then but one class, and may be symbolized by one doubly lettered circle.



Such is the relation between the class *Man* and the class *Swift Runner*, or between the class *Student* and the class *Strong Man*. To symbolize this relation the circles overlap.

(5) The classes A and B may have no member in common. Such is the relation of the two classes *Man* and *Oyster*, or of the two classes *Prime Number* and *Integral Power of Ten*. To symbolize this relation the two circles are wholly separated.



Either of two classes, then, may include the other; the two

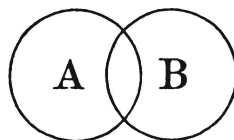
may be wholly identical, or only partially coincident, or wholly exclusive. And these five relations are, as we shall see, of much importance.*

ANOTHER METHOD OF SYMBOLIZING THE RELATIONS OF TWO TERMS.

The task of showing what are the relations of two terms, can be aided by another use of the circle-diagrams. The utility of this second method will be considerable when we come to analyze the meaning of groups of statements. Suppose, as before, that A and B are represented by circles. Suppose also, for a moment, that A and B are classes that are partially coincident, as are the classes *Man* and *Swift Runner*; then con-

sider what the diagram (4) of the previous set of diagrams can tell us. It is easy to see, by simply observing the diagram, that the paper adjacent to and within these circles is divided into four sections, as follows: First, the portion of the paper outside of both circles; second, the portion inside of A, but outside of B; third, the portion inside of B, but outside of A; fourth, the portion of the paper that is at once inside of A and of B. Now, if in A we suppose included all the objects called A, and only these, then all the paper outside of A may be supposed to include the objects in the world that are not to be called A. If A means *man*, then the paper outside of A may be supposed to be occupied by birds, triangles, prime numbers, lizards, republics, stars, angels, and all other things, of whatsoever nature, that agree in this, viz., that they are not *men*. Even so all the paper outside of B is occupied by whatever is not B. This being understood, the four sections of the paper will stand for four distinct classes of objects, as follows: First, the objects in the world that are neither A's nor B's; second, the objects in the world that are A's, but not B's; third, the objects in the world that are B's, but not A's; fourth, the objects in the world that are both A's and B's.

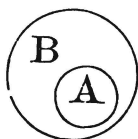
Into these four classes, be it observed, the whole world is exhaustively divided; any object that you may mention falls into one of these four classes. Conversely, if two classes, A and B, are



* The foregoing improved method of applying the old-fashioned circle notation, with the further application of the same method in the next chapter, is known to me from Lange's *Logische Studien*: cf. Mr. Venn's *Symbolic Logic*, ch. I, especially p. 30. See also Ueberweg, *Logik*, 4te Aufl., pp. 112 and 177.

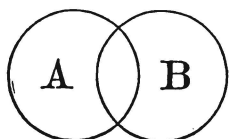
given, such that I can divide the world into four distinct classes, each as real as A and B, and if these four classes are, (a) things neither A nor B, (b) things A and not B, (c) things B and not A, (d) things both A and B: then I may be sure that the classes A and B are, as in the diagram, partially coincident and partially non-coincident.

Hence, as the student may convince himself by trial, *if A and B have to each other any other relation than that shown in this last diagram, one at least of the four classes of objects just enumerated must vanish.* For example, if A is wholly



within B, then the world is completely divided into three instead of into four classes. The three classes are: First, things that are neither A nor B; secondly, things that are B but not A; thirdly, things that are both A and B. The class of things that are A, but not B, vanishes.

In the same way, if A and B are identical, then the class of objects that are A, but not B, vanishes; as does also the class of objects that are B but not A. If B is included within A, then the class of things that are B, but not A, vanishes. If B and A are wholly separate, then vanishes the class of things that are both A and B.



As a consequence, if we agree to represent the vanishing of any one of the four classes in the annexed diagram by shading the section of the diagram that represents the class, then all five of the before-mentioned relations of A and B can be expressed by shading the appropriate sections. If I shade the part inside of A, but outside of B, then I indicate that the section of A's that are not B's, is to be destroyed; *i. e.*, that A is either wholly identical with B, or wholly within B. If I then wish to indicate that A and B are in fact identical, I must shade the part of the diagram inside of B, but outside of A. And so I should treat the other cases.*

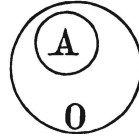
EXAMPLES OF CLASS RELATIONS.

We may exemplify the foregoing methods of notation by a few concrete examples. Take the classes *Animal* and *Organism*.

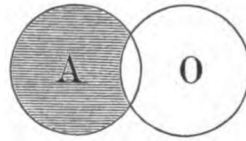
* This method of using the circle-notation is Mr. Venn's; cf. *Symbolic Logic*, ch. V.

Their relation may be symbolized in either of the two ways just described. We know that animals are included among organisms, and that there are other organisms besides the animals. Hence the circle organism will in one notation include the circle animal, while, in the other notation, the two circles will intersect, but out of the four classes indicated by the intersection, one will be caused to disappear by shading it.

FIRST METHOD.

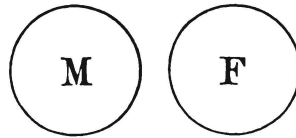


SECOND METHOD.

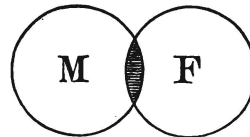


FIRST METHOD.

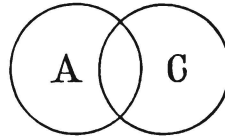
As a second example, take the classes *Mammal* and *Fish*. The two ways of representing their relations appear in the adjoining figures. Both diagrams express the same fact in different ways.



SECOND METHOD.



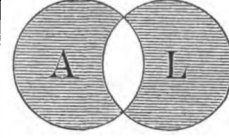
As a third example, take the classes *American* and *Caucasian*. In this case the two methods of symbolizing the relations of the two classes, lead us to the same figure.



FIRST METHOD.

SECOND METHOD.

As a fourth example, take the classes *Equiangular Triangle* and *Equilateral Triangle*. Call the corresponding circles A and L. The two methods of symbolizing the class-relations will lead us to the adjoining figures.



ANOTHER NOTATION: IMPORTANCE OF THE SUBJECT.

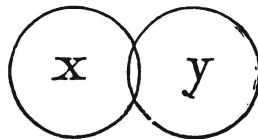
The importance of thus symbolizing the relations of two classes by some simple and exact kind of notation, will hereafter appear very prominently. But even now the student can see the use of

such notation as an aid in the clear conception of the relative meanings of two words. When asked to compare two synonyms, such as *difficulty* and *obstacle*, or as *assumption* and *presumption*, or as *belief* and *credence*, we go about our work much more easily if we have a definite notion of the kind of comparison we are to make. Now to compare the meanings of two words, A and B, is for the first simply to find out whether anything that is called A is also called by the name B; and if so, whether there is any A that is not called by the name B; and, finally, in case everything called A is also called B, whether the names A and B have identical import, or whether there are things called B but not A. The answers to these questions are clearly and simply expressed by either of the foregoing circle-notations; and the fact that we know how to express the relative meanings of two words by the relative position of two circles, is of great aid to us in gaining a simple and fixed notion of what it is to compare the meanings of two words.

The importance of fixing the habit of regarding terms as names for any one of a collection of individual objects, and the use that we shall hereafter make of this way of regarding terms, both lead us to suggest yet another notation for the relations of classes and of the corresponding terms. This notation will be algebraic, and as far as it goes will be essentially the same as that introduced by Boole in his *Laws of Thought*, published in 1854.

Let the letters of the alphabet stand for any classes you choose. Thus let x be the name of some class of objects. Then the extension of x will be the sum total of the things in the world that are called by the name x . Now let all the things that you choose to take note of, except those that are called x , be considered as grouped together into one class. The most appropriate name for this class will be *not- x* . Symbolize the class *not- x* by a horizontal mark drawn over x , thus, \bar{x} . Then all the objects that you may choose to consider in the whole world will be divided into two classes, viz., into x and \bar{x} , just as before all things were divided by the circumference of a circle into two parts, the one representing a class, the other whatever might exist beyond that class. If x means horse, then cows and conquerors and steam-engines, and whatever other class you may choose to compare with the first, will belong to \bar{x} . In the same way let y name another class.

Then \bar{y} may name the things that are *not-y*. But now what are the possible classes that may be formed by dividing all the objects of the world according to the two classes x and y . Evidently, as the circle-diagram will show, the world may have four classes in it, viz.,



The objects that are x and y .

The objects that are x and \bar{y} .

The objects that are \bar{x} and y .

The objects that are \bar{x} and \bar{y} .

Perhaps some one of these four classes, perhaps some two of them, may be found to be non-existent or impossible. But for the first all four are possible, just as the intersection of the circles x and y , until we know the facts about the relations of the circles, must be regarded as possible.

Now, for the sake of convenience, let us assume the following principles of notation: The class of things that are at once within each of two given classes, shall be symbolized by writing together the symbols of the two classes, just as in common algebra the multiplication of a and b is symbolized by writing a and b together, as ab . Thus xy shall mean "the x 's that are also y 's"; $\bar{x}y$ shall mean "the *not-x*'s that are also y 's." If x means *Chinese*, and y means *laundryman*, then xy shall mean *Chinese laundryman*; $\bar{x}y$ shall mean *laundryman that is not Chinese*; and so on. The order in which the symbols are written shall be in this case indifferent, so that xy shall mean the same as yx . The symbol $=$ shall be used to connect class symbols that stand for *identical classes*; so that we shall write: $xy = yx$; $\bar{x}y = y\bar{x}$; $\bar{y}x = x\bar{y}$; and so on of all combinations. The symbol 0 shall be used to mean *no class*, or, *nothing at all*. The equation $x = 0$ shall hence mean that the class x does not exist, or that in the possible world there is no such class. The expression $x > 0$ shall mean, on the other hand, that the class x does exist. We shall then be able to express the following facts: With respect to the classes x and y , the world may be divided into four possible groups of objects, symbolized by the combinations, xy , $\bar{x}y$, $x\bar{y}$, $\bar{x}\bar{y}$, or equally well by the precisely equivalent symbols, yx , $y\bar{x}$, $\bar{y}x$, $\bar{y}\bar{x}$. But in particular cases one or more of these groups may vanish, as we before saw in

studying the circle-notation. Whatever one of them vanishes may be equated to 0; and by this means, and by the use of the symbol $>$, any one of the possible relations of two classes may be symbolized.

Thus: the classes *poor man* and *honest man* are known to be partially coincident and partially non-coincident. Not all poor men are honest, nor all honest men poor. Let x be *poor man* and y be *honest man*. Then, the world being divided into the four possible classes, xy , $x\bar{y}$, $\bar{x}y$, $\bar{x}\bar{y}$, the facts will be expressed by symbolizing that all these four classes do exist, or by writing the inequalities: $xy > 0$, $x\bar{y} > 0$, $\bar{x}y > 0$, $\bar{x}\bar{y} > 0$.

Again, the classes *just man* and *highway robber* being symbolized, the one by x , the other by y , the facts will be expressed by the notation, thus: $xy = 0$, $x\bar{y} > 0$, $\bar{x}y > 0$, $\bar{x}\bar{y} > 0$: since both the classes x and y are actually existent. And thus whatever is expressible by the circle-notation is expressible by this notation.

The student may be helped by seeing all three ways of comparing two terms placed in close juxtaposition. This is done in the following table:

Relation Expressed.	Circle Notation. First Form.	Circle Notation. Second Form.	Algebraic Notation.*
1. Identity.			$xy > 0$ $\bar{x}y = 0$ $x\bar{y} = 0$ $\bar{x}\bar{y} > 0$ or $= 0$
2. Perfect Inclusion.			$xy > 0$ $\bar{x}y > 0$ $x\bar{y} = 0$ $\bar{x}\bar{y} > 0$ or $= 0$
3. Perfect Inverted Inclusion.			$xy > 0$ $\bar{x}y = 0$ $x\bar{y} > 0$ $\bar{x}\bar{y} > 0$ or $= 0$
4. Partial Inclusion.			$xy > 0$ $\bar{x}y > 0$ $x\bar{y} > 0$ $\bar{x}\bar{y} > 0$ or $= 0$
5. Total Exclusion.			$xy = 0$ $\bar{x}y > 0$ $x\bar{y} > 0$ $\bar{x}\bar{y} > 0$ or $= 0$

* Notice that we must take account in certain cases of the possibility that either x or y should include *everything possible*, or of the possibility that $x\bar{y}$ or $y\bar{x}$ includes everything.

§ 14. GENUS, SPECIES AND DIFFERENCE: DIVISION.

When one class includes the whole of another, the smaller class is said to be, in a broad sense of the term, a species of the larger. Thus *goose* is a species of *bird*, *sophomore* is a species of *student*, *elephant* a species of *mammal*, *blue* a species of *color*, and so on. The larger class is then said to be a *genus* with respect to the smaller.

When two classes are related as *genus* and *species*, the genus has the greater extension; the species the greater intension. The species-term means or connotes more properties; the genus-term applies to more possible things. Thus *Frenchman* means or connotes more qualities than *man*, but applies to or denotes fewer objects.

The connotation of the species-term differs from that of the genus-term by the addition of some new property. Thus, *man* connotes more than does *animal*, i. e., adds to all the properties connoted by *animal* the property *rationality*; for men are rational animals. Again, *Frenchman* connotes all that *man* does, but adds the new properties implied in the qualifying word *French*. The new property that must be added to the connotation of the genus in order to complete the connotation of the species, is called the *Difference*. The difference indicates the respect in which all the members of the species *differ* from all the other members of the genus. Thus, men differ from all other animals in being rational. Frenchmen differ from all other men in being French.

GENUS AND DIFFERENCE USED IN DEFINITION.

If we already know or have defined what any genus (e. g., *animal*) means, and if we then mention the name of this genus, and add thereto the mention of some property (e. g., *rationality*) as the difference that shall distinguish some species (e. g., *man*) of the genus from all the other members of the genus, then we shall have sufficiently defined this species itself. And hence one rule of definition, simple in appearance, but not always very easy of application, is this: *To define any class of objects, find some known genus to which they belong, and then name the property that distinguishes them from all other members of the mentioned genus.* Or, again, a fair definition is found when we can mention an

already known genus and a plain difference, that together constitute the connotation of the word we are to define. So man is defined as a rational animal, plane triangle as a plane figure having three sides, and so on. But, as said before, we are less concerned here with the methods of definition than with the illustration of the nature and the possible relations of terms.

DIVISION.

Since a genus is larger in extension, or is a larger class than one of its own species, the genus may be considered as divided in respect to any one of its species into two parts. Of these one part comprises the whole of the species. The other part comprises all the possible members of the genus that do not belong to this species. Or, using our previous notation, if x names the genus and y one of its species, then the genus x is divided with respect to this species into two classes, xy and $x\bar{y}$. Or again, if we use the symbol $+$ to indicate that the extensions of two terms are added together, or that a new class is made by uniting all the objects named by the one term and all the objects named by the second term into one group, then we may write the equation, $x = xy + x\bar{y}$.

The equation $x = xy + x\bar{y}$ means then that x is exhaustively divided into the two classes, the x 's that are y and the x 's that are not y .

The operation of division may be in form yet more generalized. Any class may be exhaustively divided with respect to any other class whatever, be this other class a species of the first or not. Only then, in some cases, one of the members of the division will vanish, either from the world of possibility or from our real world, and the division will turn out, not a false one, but a useless one. Thus I can divide *horses* into *horses that have wings* and *horses that have not wings*. But the class of *horses that have wings* vanishes from our real world. In itself such a class is possible. As a fact it does not exist. Let x and z be any classes whatever: then $x = xz + x\bar{z}$. But, for all we know, one of these species of x may vanish, *i. e.*, may be impossible. If so, we shall find out the true relation of the classes x and z . If $xz = 0$, then we have $x = x\bar{z}$, which means that the classes x and x that is not z are

identical. This result can be true only in case the classes x and z are wholly exclusive of each other. If $\bar{x}z = 0$, then we have $x = xz$, or the class x is precisely identical with the class x 's that are z . This result must mean that x is either identical with z or wholly included within z . The student can easily express these results in the circle-notation.

Having given the division of x into x that is z and x that is not z , we can further divide each of the two classes xz and $x\bar{z}$ according to some other class, as u . Either xz or $x\bar{z}$ may be regarded as consisting of u and not u . Thus, $xz = xzu + x\bar{z}u$. And $x\bar{z} = xzu + x\bar{z}u$. Hence we have a further division of x , thus: $x = xz + x\bar{z} = xzu + x\bar{z}u + xzu + x\bar{z}u$. And so we might continue indefinitely.

Thus then we could divide the class *animal* with respect to the two classes *mammal* and *swift-footed* thus:—*Animals* are as a class, divisible into (1) *animals that are both swift-footed and mammals*, (2) *animals that are swift-footed but not mammals*, (3) *animals that are not swift-footed and that are mammals*, (4) *animals that are neither swift-footed nor mammals*. If x stands for animal, y for swift-footed, and u for mammal, the above equation will precisely express this division.

A general rule can now be given for dividing any class x with reference to any number of other classes, p, q, r, s, t , etc. This rule will be plain if we mention the following principles: (1) Any class as x , divided according to any other class as p , falls into two possible sub-classes; divided according to two other classes, *e. g.*, p and q , the first class falls into four possible sub-classes; divided according to three other classes, p, q and r , the first falls into eight sub-classes, and so on. (2) These classes are shown in the annexed table, where the division of x into sub-classes is given for two, for three, for four and for five other classes. (3) Any one of the possible sub-classes may disappear if the relations of the terms are such as to make it an impossible sub-class. Knowing that any one of the sub-classes does disappear, we know something of the relations of the mentioned classes.

TABLE.
x may be divided according to

<i>p</i>	<i>p</i> and <i>q</i>	<i>p</i> , <i>q</i> , and <i>r</i>	<i>p</i> , <i>q</i> , <i>r</i> , and <i>s</i>	<i>p</i> , <i>q</i> , <i>r</i> , <i>s</i> , and <i>t</i>
into	into	into	into	into
$x \left\{ \begin{array}{l} p \\ \bar{p} \end{array} \right.$ <i>2 Classes.</i>	$x \left\{ \begin{array}{l} pq \\ p\bar{q} \\ \bar{p}q \\ \bar{p}\bar{q} \end{array} \right.$ <i>4 Classes.</i>	$x \left\{ \begin{array}{l} pqr \\ pqr \\ pqr \\ pqr \\ pqr \\ pqr \\ pqr \\ pqr \end{array} \right.$ <i>8 Classes.</i>	$x \left\{ \begin{array}{l} pqrst \\ pqrst \\ pqrst \\ pqrst \\ pqrst \\ pqrst \\ pqrst \\ pqrst \end{array} \right.$ <i>16 Classes.</i>	$x \left\{ \begin{array}{l} pqrst \\ pqrst \\ pqrst \\ pqrst \\ pqrst \\ pqrst \\ pqrst \\ pqrst \\ pqrst \\ pqrst \\ pqrst \\ pqrst \\ pqrst \\ pqrst \\ pqrst \end{array} \right.$ <i>32 Classes.</i>

It is apparent from the above that when we thus divide any class into sub-classes, the sub-classes exclude one another; and, further, that if the sub-classes be taken altogether, they will precisely equal the class from whose division they spring. Thus, if I divide mammals into carnivorous and not carnivorous mammals, no carnivorous mammal will be found among the not carnivorous mammals, and the carnivorous and not carnivorous mammals taken together, will precisely make up the class mammals.

NOTE ON THE WHOLE CHAPTER.

We have treated terms in the foregoing as if they were names with a fixed intension and extension, as if no ambiguity interfered with our understanding of words. We have been treating *ideal terms*. But the words of our common speech are nearly all ambiguous or *equivocal*. Yet each meaning of each whole term-

word must be capable of expression in a form as simple and rigid as that in which the ideal terms of this chapter have been expressed. If, then, one is ever in doubt about the meaning of a term-word, let him remember that he may not rest until he has found so definite a meaning for this word as to render possible an exact statement of the relation between the class of possible objects denoted by the word in question, and some other already known class of objects. Nothing less will in the least suffice. X is not known to me as a class all alone. X is known to me as in definite relation to some other class Y . Through the difference of these two classes I learn to know each better. As a practical direction, then, let the student bear this in mind: To be sure you understand language, repeatedly compare various related terms together, to see just how the classes named by them are related. Use often for this purpose some form of notation. By persisting in this habit you will free yourself from the danger of being constantly led captive by ambiguous words.

EXERCISES ON TERMS AND THEIR RELATIONS.

1. Tell concerning each of the following terms what kind of object it names: thing, quality, action, state, or relation. Also, whether it names a single object by itself, or any one of a class of objects.

Picture	Ferocity	Protestantism
Hamlet	Square Root	Monarchy
Quiescence	Omnipotence	Ability
Motion	Reality	Size
Logarithm	Flowing	Fidelity
Dentistry	Trend	Wilderness
Insufficiency	Talent	Alcoran
Buddhism	Jury	Army
Astrology	Anger	Johnsonian English
Equality	Liberty	Nearness
Far	North	Death

Note on the foregoing list: A term may name a *collection* of things or a *group* of qualities, or of relations, or of states, or of actions, and may name this collection as a single object. Such a term is *crowd*. The student should classify these *collective names* under the head to which belong the members of the group that they name. Collective names may name single collections, or classes of collections. If any of the foregoing words are ambiguous, let the student classify them in each of their various uses.

2. Give the relations of the following pairs of classes, and symbolize each relation by all three notations: *Steam* and *vapor*; *life* and *death*; *exclusion* and *seclusion*; *obstacle* and *difficulty*; *elephant* and *useful animal*; *knowledge* and *belief*; *strength* and *power*; *incapacity* and *inability*; *offense* and *defense*; *liberty* and *license*. In case of ambiguity in the foregoing terms, take each term in all of its important

meanings, and compare each meaning of each term with each meaning of the other term of the same pair.

3. Divide *man* according to the three classes *Englishman*, *author*, and *critic*, into all the possible sub-classes. Divide *quadruped* according to the four classes *strong*, *hungry*, *carnivorous*, and *rational*; and point out what sub-classes disappear from this division upon the supposition that men alone are rational animals. Divide *number* according to the three classes *prime number*, *even number*, *number less than one hundred*, and show what classes vanish from the list of classes as soon as you consider their definitions.

Note: To save time, symbolize the foregoing classes by appropriate letters, and use the table of divisions.

CHAPTER V.

THE DETERMINATION OF THE MEANING OF STATEMENTS. (§§ 15-18.)

§ 15. GENERAL CONSIDERATIONS.

We now come to the most important part of our work. At the outset (*v.* § 3), we declared our work to be: *The systematic analysis of the meaning expressed and implied in any given sentence, or collection of sentences.* To perform this task we first investigated, in §§ 5-10, the nature of sentences as expressions of assertions; and we reduced sentences to uniform typical statements, or to series of such typical statements. The typical statement was an expression, though not a perfectly simple or exact expression, of the meaning of language. We observed, however (§ 7), that the typical statement always tells us *that two names have or have not, in whole or in part, the same application.* This general observation would be itself of some aid in establishing the exact meaning of any statement. But we had to study yet further the names or terms themselves. In the last chapter we therefore considered the relations that two names can bear to each other. The relations, as there distinguished and classified, were definite relations. The question arises: If we take once more the typical statement *A is B*, does this statement tell us definitely the relation between the two classes *A* and *B*? That is, does this statement tell us which one of the relations expressed in the table of the relations of terms (*v.* § 13) is the relation existing between *A* and *B*? On the answer to this question depends our understand-

ing of statements. Our business, therefore, is now this: We have reduced statements to the typical form, and are to investigate further the meaning of the typical form itself.

a. THE FOUR TYPICAL STATEMENTS.

In § 9, *d*, we divided logically simple statements into the classes represented by the three forms: *A is B* (*A* being now the name of a *single* object). *Some A's are B's*; *all A's are B's*. Each of these affirmative forms had its corresponding negative. If we now consider the meaning of these statements, it will be noticed that the first and third forms, while differing from each other in the character of their subject, agree in bringing the *whole class* named by the subject into some relation to the predicate class (for *A* used as the name of a single object, *e. g.*, *Socrates*, can be viewed as a class that has but one possible member). We may, then, treat the first and third forms together, while we ask the question: Into what relation to the predicate do these forms bring the subject? And to ask this question will be, as we have just seen, to ask ourselves the real meaning of these statements. Let us therefore, treat the Singular statement as an Universal statement, in all our further discussion of the meaning of statements. We shall have, then, to inquire into the meaning of the Universal and Particular forms of statement, and each of these again will have both an affirmative and a negative form. For the sake of brevity we shall symbolize these four forms by four letters, long since fixed upon by tradition for this purpose:

A=Universal Affirmative Statement: *All x is y.*

E=Universal Negative Statement: *No x is y.*

I =Particular Affirmative Statement: *Some x is y.*

O=Particular Negative Statement: *Some x is not y.*

To these four forms, then, or to groups of these four forms, all sentences containing statements can be reduced (since all singular statements are now to be classed under A). We are now to discuss the meaning of each of these four typical statements. To discuss the meaning of each form is, we have said, to show what definite relations among its terms are in it implied. After having discussed each form separately, we shall pass on to discuss groups of statements.

b. INFERENCE, OR REASONING.

The task we have now before us is nothing more or less than an account of the whole subject of *inference* or *reasoning* in so far as the process of *inference* or *reasoning* is or can be expressed in language. To infer is to pass in the mind from one assertion to another, with a consciousness that one's belief of the first assertion warrants him in believing the second. If I say to myself, *that is a steep hill*, and then think, *I shall, therefore, grow weary while climbing it*, I infer. I pass from the first assertion to the second, and hold the first assertion good ground for believing the second. If I say to myself, *the angles A and B are both equal to C, and hence to each other*, I infer. If I say, *the ground is wet, therefore it must have rained*, I infer. Now the subject of inference or reasoning, as a mental process, the discussion of its nature, its validity, its use, its forms, belongs to Philosophic Logic, and lies beyond our scope. But men express inferences in language. They say, the statement M is true, and hence the statement N must be true. Now when are men able to say this? In two cases and in two alone. If N be inferred from M, both being statements, then either M contains or implies N as a part of its own meaning; as the statement *all men are mortal*, includes as a part of its meaning the statement, *some men are mortal*: or else the statement M is first to be connected with some other statement or statements understood, and then the statement N is contained in M plus the other statements, as a part of their combined meaning. So the statements *John is angry* and *James is angry* contain as part of their combined meaning the statement: *More men than one are angry*. *A statement is inferrible from another statement, or from a group of other statements, only when it is contained in them as a part of their meaning*. If inferences are expressed in language, they therefore become a legitimate subject for our elementary logical analysis. For all that we have to do in order to tell whether a new statement of any sort whatever can be inferred from a given statement, or from a given series of statements, is: first to develop and to express the full meaning of the statement, or of the whole series of statements, and then to compare with this developed meaning the statement that is to be inferred. If this new one is contained in the previous

statements as their meaning stands thus developed, then the new statement can be inferred from them; but if not, then no such inference is possible from these statements. Furthermore, the question whether any statement stands to a previous statement in any one of the relations mentioned in § 1, can always be reduced to a question whether some statement is inferrible from this previous statement. The inquiry into the complete meaning of statements is, therefore, an inquiry into all the possible ways by which inference or reasoning can be expressed in language.

Students often look upon any discussion about the reasoning process as of necessity a very dark discussion of mysteries. No doubt there is much about reasoning, as a process of the mind, that is at present very obscure and difficult. But to discuss the reasoning process, as expressed in language, is a comparatively simple matter. To see that one statement gives the reason for another, is simply to see that the first *means* the second, or *contains* the second as a *part of its own meaning*. Reasoning is, for our purpose, just the act of saying what a given statement means, or what several statements together mean.

c. THE DIFFERENCE BETWEEN INFERENCE AND INTERPRETATION.

If what we have said be true, there is no essential difference between inference, as expressed in language, and the interpretation of language. For the sake of convenience, we shall, however, make the following distinction: By *mere interpretation* of speech, we shall mean the substitution of synonymous words for given words, and whatever else *produces statements that have for their subject and predicate terms denoting the same classes and connoting the same qualities as are connoted and denoted by the terms of the original statement*. Thus, the process of reduction to the typical form is mere interpretation, not inference proper. By *inference, as expressed in language*, we shall mean whatever *produces statements that differ from the original ones, in having for their subjects terms of different connotation and denotation from the denotation and connotation of the original terms, while the new statements are still implied in the first as a part of their meaning*. Thus it is inference to say that *since some Englishmen are stubborn, some stubborn people must be Englishmen*.

The boundary line between inference and mere interpretation is, however, very blurred.

The vast importance of correct reasoning is everywhere admitted. Possibly the student may value our present task more when he reflects that correct reasoning, expressed in language, is merely a higher form of the interpretation of speech.

d. THE EXISTENCE OF CLASSES, AND THE MEANING OF THE COPULA.

A statement of any one of the four forms tells us something of the relation between the classes *X* and *Y*. But, by § 8, *a*, the copula of a statement does not tell us anything about the existence of the objects spoken of in the statement. And so, as it seems, statements are to tell us what relations two classes bear to each other, without letting us know whether there actually exist such classes of objects as the terms name. This seems puzzling; and it is an essential part of our task to explain such obscurities as we find affecting our understanding of speech. The following considerations may help to make the present matter clearer: * (1) Although the copula does not, as copula, tell us whether or no the subject and predicate are really existent classes, yet in many sentences that contain the verb *to be*, this word acts both as copula and as verb of existence. Thus, if I say, *Socrates is ill; this horse is swift; an inscribed polygon is less than the circumscribed circle*, I seem to imply in each case the existence of subject and of predicate. But here the statement is really composite, and needs to be expanded into two parts: *e. g.*, *Socrates is a now existing being, and this being is an ill being.* (2) When in any way I affirm or imply the existence of the subject and predicate, the context must show what kind of existence I attribute to both. If I am telling about fairy-land, and say, *there exist fairies with six wings*, then the existence that I mean is a feigned existence in my fancy. If we say, *there exist only five regular polyedra*, the existence mentioned is existence in space for our power of geometrical imagination. We can conceive no more. If we say, *there is a city called San Francisco*, then our statement refers to the sort of existence with which we have to do in our every-day

* Cf. on this subject the views of Sigwart, *Logik*, bd. I, p. 94; Ueberweg, *System d. Logik*, 4te Aufl., pp. 167, 168; Mr. Venn, *Symbolic Logic*, ch. VI. Every one is here independent perforce, since authority is very conflicting.

experience. (3) But from all the various sorts of *existence* must be distinguished the *validity* that a typical statement X is Y has, apart from the thought of any sort of existence other than the *possible existence* of the objects or classes named in this statement. If I say, *all x is y* , I mean that even though there exist no x 's in the real world or in any feigned world, still x and y are so defined, have such a connotation, that *if any objects were to be found* to correspond with the names, *all the x -objects would be found inside the class of y -objects*. When I say *no x is y* , I mean that whether or no there do exist any x 's or any y 's, in this or any other world, x and y are so defined, have such a connotation, that their extensions, if they have any, do not coincide in any part. And so with the other forms.

The circles with which we have symbolized the relations of classes may be applied to the expression of statements. But then it must be understood that the circles do not stand for collections of real objects merely, but for collections of all the possible objects that, by the definition of the term, could have the name of the term applied to them. The circle called x means *the collection of all the real and possible objects that could have the name x* . Whether there are or ever will be such objects as x , we do not say in any typical statement about x , excepting only in those statements that have for their predicate the term *existent thing*. We tell in all other typical statements only about the purely ideal classes to which the names x and y apply.

When, therefore, we say $xy=0$, or $x\bar{y}>0$, or when we express these symbols by saying, *the class $x\bar{y}$ exists*, or *the class xy does not exist*, we must be understood to mean *possible existence only*, not actual existence. If I say, *no horses are winged* or *winged horses* $=0$, my statement does not tell whether or no there actually exist *horses* or *winged animals*. My statement only says: Such is our present definition of *horse* and of *winged animal*, that from the class *winged animal* we exclude the class *horse*. So much, then, for the explanation of the meaning of typical statements. The statements made in common language are thus divided into two kinds: (1) Statements that have as predicate the class *existent thing*; and (2) statements that have as predicate any other class. Statements of this second kind tell us nothing about the real existence of the classes that they compare, but only about

the relations of these classes viewed as collections of possible objects making up the extensions of their corresponding terms. To say that the class x includes the class y , is to say that there are possible more things that could be named x than things that could be named y .

§ 16. THE WHOLE MEANING OF A SIMPLE STATEMENT REFERRED TO ITS SUBJECT- AND PREDICATE-TERMS AND TO THEIR NEGATIVES.

We shall now consider simple typical statements of the four forms, A, E, I, O. In case of each form we shall set forth the whole meaning of a single statement of that form, considered with reference to its subject- and predicate-terms and to their negatives. Thus, we shall take the form A, *All x is y* . We shall consider what this one statement, viewed by itself, tells us about x , about y , about *objects not x* (or \bar{x}), and about \bar{y} . So we shall do with the other statements, arranging our results in some convenient order.

a. CLASS-RELATIONS EXPRESSED IN A, E, I, and O.

In order to find what A, E, I, and O respectively tell us about the relations of their terms, let the student first look back at the table of class-relations in § 13. We shall hereafter refer to the various relations in that table by their numbers. Thus relation 1 shall mean the relation of identity between two classes, and so on. The class A in that table shall now be called x , the class B shall be y . To say x bears to y relation 2, shall mean that x is included within y . To say x bears to y relation 3, shall mean that x includes y . For the present let attention be fixed on the first column of the table.

A: *All x is y* , tells us that x is wholly included within y . But x may be a part of y or the whole of y . Thus, in the statement *all men are mortal*, men are only a part of the class *mortal*, which includes many other actual and possible classes. In the statement, *all equilateral plane triangles are equiangular plane triangles*, the subject and predicate can be proved to be identical classes, A therefore leaves it undecided whether the relation of x to y is 1 or 2. But the relation of x to y must in A be either 1 or 2, and cannot be 3, 4 or 5.

E: *No x is y* , tells us that the relation of x to y is relation 5, and can be no other.

I: *Some x is y*, leaves it quite undecided whether the relation of *x* to *y* is 1, 2, 3 or 4, but excludes the relation 5. Thus in *some equilateral triangles are equiangular*, the relation of subject to predicate is relation 1. In *some men are mortal*, the relation is 2. In *some mortals are men*, or *some winged creatures are birds*, the relation of subject to predicate is 3, where *x* is the subject-term. In *some men are strong animals*, the relation is 4. But never in case of I could the relation be 5.

O: *Some x is not y*, leaves it undecided whether the relation of *x* to *y* is 3, 4 or 5, but excludes 1 and 2. Thus in the statement *some winged beings are not birds*, where *winged beings* = *x*, and *birds* = *y*, the relation is 3. In *some men are not strong animals*, the relation is 4. In *some sheep are not carnivorous*, the relation of subject to predicate is 5.

b. THE CLASS RELATIONS STATED CONVERSELY.

To tell how the predicate of one of these statements is related to the subject, is now an easy task. But to do this is to answer the question, "From *all x is y*, what can we infer about *y*?" or is to answer the questions "From *some x is y*, or from *some x is not y*, or from *no x is y*, what can we infer about *y*?" To answer these questions is to discover the *converse* of each of the above statements.

Let us notice the following facts:

If <i>x</i> bears to <i>y</i> relation 1	then <i>y</i> bears to <i>x</i> relation 1.
If <i>x</i> " " <i>y</i> " 2	" <i>y</i> " " <i>x</i> " 3.
If <i>x</i> " " <i>y</i> " 3	" <i>y</i> " " <i>x</i> " 2.
If <i>x</i> " " <i>y</i> " 4	" <i>y</i> " " <i>x</i> " 4.
If <i>x</i> " " <i>y</i> " 5	" <i>y</i> " " <i>x</i> " 5.

These facts are evident on mere inspection of the circle-diagrams of the first column of the table in § 13.

Hence, since A leaves it doubtful whether *x* bears to *y* relation 1 or relation 2, the predicate *y* of A may bear to *x* the relation 1 or the relation 3, and we know not from A which is the true relation of *y* to *x*, whether 1 or 3.

In like manner, since E tells us that *x* bears to *y* relation 5, the predicate *y* of E must bear to *x* the relation 5.

Since I tells us that the relation of *x* to *y* may be 1 or 2 or 3 or 4, but cannot be 5, we can infer as the meaning of I that the predicate *y* may bear to *x* one of the relations 1 or 3 or 2 or 4, but cannot bear the relation 5.

Since O tells us that the relation of x to y may be 3 or 4 or 5 but cannot be 1 or 2, y , the predicate of O, may bear to x one of the relations 2 or 4 or 5, but not the relations 1 or 3.

Hence, to summarize :

Proposition.	Relation of x to y .	Relation of y to x .
A	(1) or (2).	(1) or (3).
E	(5).	(5).
I	(1), (2), (3), or (4).	(1), (3), (2), or (4).
O	(3), (4), or (5).	(2), (4), or (5).

c. CONVERSE STATEMENTS.

The single statement that expresses in one of the forms A, E, I, or O the possible relation of the predicate of a former statement to the subject of that statement, is called the converse of the former statement. The subject of the former statement is made the predicate, the predicate of the former statement is made the subject of the new statement.

The converse of A must be (v. the above summary) a statement whose new subject (the old predicate of A) bears to the new predicate one of the relations 1 or 3. Only a statement of the form I can leave doubtful whether its subject bears to its predicate the relation 1 or the relation 3. Hence, the converse of A must have the form I, and may be stated by writing a sentence of the form I with y for its subject and x for predicate.

For like reasons the converse of E must have the form E, and the converse of I must have the form I. The converse of O, if it existed, would have its subject bearing to its predicate any one of the relations 2, 4, or 5. A simple statement that permits the subject to bear to the predicate any one of these relations (leaving doubtful which is the real relation), cannot be found in any one of the forms A, E, I, and O. Hence the converse of O cannot be stated in any one of the simple forms A, E, I, or O. Hence, we can write:

The converse of *all x is y* is *some y is x* .
 " " " *no x is y* is *no y is x* .
 " " " *some x is y* is *some y is x* .
 " " " *some x is not y* is *lacking*.

d. RELATIONS OF NEGATIVE CLASSES IMPLIED IN A, E, I AND O.

We are next to see what each of our four typical statements tells us about the negative terms \bar{x} and \bar{y} . Here again the student must refer to the diagrams expressing the class-relations in the first column of the table of § 13. Let him now regard the space outside each class-circle as itself a figure that has its outer boundary quite indefinite, the other boundary being the circumference of the class-circle. Then it is plain that x always bears to \bar{x} the relation 5, and that y always bears to \bar{y} the relation 5. Hence, by inspection we can see that the following table is true.

Relation of	Corresponding Relation of						
x to y .	y to x .	\bar{x} to \bar{y} .	\bar{y} to \bar{x} .	x to \bar{y} .	\bar{y} to x .	\bar{x} to y .	y to \bar{x} .
1	1	1	1	5	5	5	5
2	3	3	2	5	5	4 or 2	4 or 3
3	2	2	3	4 or 3	4 or 2	5	5
4	4	4 or 5	4 or 5	4 or 3	4 or 2	4 or 2	4 or 3
5	5	4 or 5	4 or 5	2 or 1	3 or 1	3 or 1	2 or 1

Account must be taken of the case where either x or y includes everything. From this table follows forthwith this second one:

	Relation of		Corresponding Relation of					
	x to y .	y to x .	\bar{x} to \bar{y} .	\bar{y} to \bar{x} .	x to \bar{y} .	\bar{y} to x .	\bar{x} to y .	y to \bar{x} .
A	1 or 2	1 or 3	1 or 3	1 or 2	5	5	5, 4, or 2	5, 4, or 3
E	5	5	4 or 5	4 or 5	2 or 1	3 or 1	3 or 1	2 or 1
I	1, 2, 3, or 4	1, 3, 2, or 4	1, 3, 2, 4, or 5	1, 2, 3, 4, or 5	5, 4, or 3	5, 4, or 2	5, 4, or 2	5, 4, or 3
O	3, 4, 5	2, 4, or 5	2, 4, or 5	3, 4, or 5	4, 3, 2, or 1	4, 2, 3, or 1	5, 4, 2, 3 or 1	5, 4, 3, 2, or 1

This table gives us a complete account of the meaning of each of our four forms, including also the implications about negative

terms. But the results can easily be interpreted, as before, in a series of new statements that can be inferred from the statements all x is y , no x is y , etc. Such statements are:

From A (*all x is y*) can be inferred:

Some not- x is not- y (if there is any not- y).

All not- y is not- x .

No x is not- y .

No not- y is x , etc.

From E (*No x is y*) can be inferred:

Some not- x is not not- y .

All x is not- y , etc.

And so of the others.

This way of expressing our full meaning, though complete, is less convenient than the next method.

e. ALGEBRAIC NOTATION.

The statement A declares that x is wholly within y . Hence A tells us that $x\bar{y}$ is non-existent or impossible; and, furthermore, that xy is a possible class, that must be real if x and y are real. But A leaves us in doubt whether x fills up the whole of y or not.

Hence to express A by the algebraic notation, before explained, we take the equations, $xy > 0$, $x\bar{y} = 0$, $\bar{x}y > 0$ or $= 0$. We also have in general $\bar{x}\bar{y} > 0$, unless y includes all possible objects. In that case $\bar{x}\bar{y} = 0$. But this case seldom arises.

E excludes x from y , but leaves doubtful whether x is not identical with the the whole of \bar{y} . To express E we have the equations: $xy = 0$, $x\bar{y} > 0$, $\bar{x}y > 0$, $\bar{x}\bar{y} > 0$ or $= 0$.

I is expressed thus: $xy > 0$, $x\bar{y} > 0$ or $= 0$, $\bar{x}y > 0$ or $= 0$, $\bar{x}\bar{y} > 0$ or $= 0$. The second case of the last equation would be found to hold good if x and y between them named all possible objects.

O is expressed thus: $x\bar{y} > 0$, $xy > 0$ or $= 0$, $\bar{x}\bar{y} > 0$ or $= 0$, $\bar{x}y > 0$ or $= 0$.

Each one of the above statements, symbolically expressed, enables us to make some statement about y , \bar{x} , or \bar{y} . Thus, take A. Let it be required to show what A implies about \bar{y} . Of the four abstractly possible classes of things xy , $x\bar{y}$, $\bar{x}y$, $\bar{x}\bar{y}$, A tells us that the class $x\bar{y}$ does not exist, while the class xy does exist, and the other classes may exist. There is, then, *no y that is x* , and thus at once we see that \bar{y} has to x a relation expressible by the

proposition E, and to \bar{x} a relation expressible by the proposition A; and so, *all not- y must be not- x* .

Thus the interpretation of the whole meaning of a statement becomes possible. If a statement implies that some one of the four sub-classes, xy , $\bar{x}y$, $x\bar{y}$, $\bar{x}\bar{y}$, must vanish, then we can at once make a new statement about some one of the classes \bar{x} , y , or \bar{y} . For of some one of these classes we can say that it cannot be combined with one of the other classes; *i. e.*, that these two classes have no members in common.

We have begun with the statement having x for its subject, and have been investigating its whole meaning in terms of x , y , and \bar{y} . We could just as well have begun with a statement having for its subject \bar{x} , or y , or \bar{y} , and could have investigated its meaning with reference to the other terms. The plan would be the same. And so we have a general rule:

To find the complete meaning of a statement in terms of its subject and predicate, first reduce the statement to the appropriate typical form, A, E, I or O. Then by the use either of the circle notation or of the algebraic notation, state completely what are the class-relations implied in this statement. Then express these class-relations at pleasure in new statements.

We have not especially mentioned the obvious fact that *all x is y* includes as a part of its meaning, *some x is y* , and that *no x is y* , includes as a part of its meaning, *some x is not y* .

f. THE NEGATIVE TERMS.

A word may yet be added to facilitate the use of the foregoing. The negative terms \bar{x} and \bar{y} have had so far a wholly indefinite extent. They have been used to mean anything that is not x or that is not y . We may now speak a little more clearly about their meaning, since we have reached the point where the student can see the use of better definition. In our speech we very seldom are thinking of all possible objects as a whole; but we commonly have some largest class in our minds, and within this class we confine ourselves when we make statements. Thus, we talk sometimes of mankind, without thinking about the relation of mankind to the rest of the world, but confining ourselves wholly to the human race. If, now, while we are doing this, we say: *X does not belong to the class of just men*, we do not intend to class-

ify X with triangles and cabbages, and all the other things in the world that are not *just men*. We simply want to classify X with those *men that are not just*. If, then, y stands for the class *just man*, \bar{y} will stand, so long as we confine our discourse to *man-kind*, for the class *unjust man* and for no other class. The negative \bar{x} of a term x , means, then, not necessarily all possible objects of any class beside x ; but only all possible objects that are not x in the *largest class concerning which we are at the time discoursing*. If I am talking of *men*, *not-white* means a man that is not white; if of visible objects in general, *not-white* means an object otherwise colored; if of absolutely all possible things, *not-white* would include logarithms and perfumes. The meaning of negative terms varies, therefore, with the subject of discourse.

If the student will keep this fact in mind, and will reflect in each case on the meanings of the negative terms that he may use, he will now find himself quite able to state in full the meaning of any given simple sentence.

§ 17. THE MEANING OF GROUPS OF STATEMENTS REFERRED TO THE TERMS OF EACH STATEMENT AND TO THEIR NEGATIVES.

How to find the combined meaning of several statements is now our task. Let us exemplify it by one or two simple instances. Suppose I know that *some x is y* , and, also, that *no not- x is y* . What do these statements in combination tell me? Referring to the *second column* of relations in the table of § 13, I see that the second of these statements bids me strike out all that portion of the diagram that is outside of x and inside of y , while the first statement bids me regard x and y as at least partly coincident. In combination, then, the two statements tell me that there is some of x inside of y , and that inside of y there is nothing but x . Or, in other words, *all y is x* , although I do not know whether y constitutes the whole of x , or whether there may not be a part of x outside of y . To find, then, the combined meaning of several statements, I have to find out what classes of objects they together exclude from the realm of possibility, what classes they declare to be possible, and what classes they leave wholly untouched. Then at pleasure I can state the result in various forms. The second column of our table of relations is for the present purpose especially useful. Thus, if one statement shades out one part of

a diagram, and a second statement shades out another part, the combined meaning of the two statements shades out both parts at once. If *no equiangular triangles are other than equilateral*, and *no equilateral triangles are other than equiangular*, the combined meaning of these two statements can be put in the form, *equilateral and equiangular triangles are identical classes*. This inference could be symbolized by letting x be one of these classes and y the other, and by then shading out in succession the part of x outside of y , and the part of y outside of x , leaving as the relation of x and y the relation 1 of the table. In some such way now, we are to develop all the information that we get from several statements at once.

a. CONDITIONS OF COMBINATION, CONSISTENCY.

When we combine several statements to find what they together mean, we may find that two of them are *inconsistent*. Inconsistent statements are those that mean or that imply incompatible relations of classes. Such statements cannot be combined. Thus the statements, *all x is y* , and *some not- y is x* , are inconsistent, and together give no meaning. The one demands that we shade out the class $x\bar{y}$; the other declares this class to be still in existence; the one says $xy=0$, the other, $xy>0$. To find whether two statements are inconsistent, we expand their full meaning, as in the last section. Then we see whether any part of the meaning of one puts two terms into a relation incompatible with a relation that is included in the meaning of the other.

When two statements are inconsistent in the particular way called *contradiction*, the statements are such that if either of them is false, the other is true. Thus, A and O, if they have the same subject and predicate, are contradictory. For A states that x , the subject, has to y , the predicate, either the relation 1 or the relation 2. O states that x has to y , one of the relations 3, 4, 5. Now A and O must be inconsistent, for x and y cannot have two relations at once. And if x has to y relation 1 or relation 2, it cannot have any of the relations 3, 4, 5; and if x has to y either the relation 3, or the relation 4, or the relation 5, it cannot have to y either the relation 1 or the relation 2. Hence, if A is true, O is false, and if O is true, A is false. In like manner E, which states that x stands to y in relation 5, contradicts I, which excludes 5, but states that x stands to y in one of

the relations 1, 2, 3, 4. But A and E, though inconsistent, are not contradictory. For according to A, the relation of x to y is 1 or 2. According to E, the relation of x to y is 5. But both may be false, and the true relation may be 3 or 4, expressed either by I or by O. I and O are consistent, because they both permit x to have to y the relations 3 and 4. But if both I and O are true at once, 1 and 2, which I would leave possible if it were alone true, and 5, which O would leave possible if it were alone true, are excluded.

Combination may have as its result some new statement, as was the case in our previous example. But sometimes the combination of two statements gives us nothing new, but merely sums up the meaning of both statements in a form not even abbreviated. Thus the two statements: *All men are mortals*; *all planets revolve in elliptic orbits*, can give us no new result. For the two statements together involve four distinct classes; of two of these, one statement tells us the relations; of two more, the second statement gives us the relations. But of the relation of the first to the second pair, we learn nothing. In general, for the purposes of the sort of inference that we study in this book, it is true that any number n of statements, if they involve together more than $2n$ independent terms, give us as joint inference, when they are taken together, nothing that we could not have inferred from them separately. The exceptions to this rule are beyond our present scope. By independent terms we mean, of course, such terms as x and y , not such terms as x and \bar{x} .

b. METHOD OF COMBINATION.

By a simple example we may now make clear how we can proceed to develop systematically the joint meaning of several statements referred to their subject and predicate-terms. Let us, for a moment, drop symbols, and take as our example a concrete case.

We have given us the statements: *All Greeks are brave* and *all Greeks are dwellers by the sea*. We are required to develop the full meaning of these two statements in terms of their subjects and predicates. That is, we are required to state in full the inferences that can be drawn from these two statements. By § 14 the world of mankind is, in the abstract, or before we know anything further, divisible into eight possible sub-classes:

1. Greeks, that dwell by the sea, and that are brave.
2. Greeks, that dwell by the sea, and that are not brave.
3. Greeks, that do not dwell by the sea, and that are brave.
4. Greeks, that do not dwell by the sea, and that are not brave.
5. Not-Greeks that dwell by the sea, and that are brave.
6. Not-Greeks, that dwell by the sea, and that are not brave.
7. Not-Greeks, that do not dwell by the sea, and that are brave.
8. Not-Greeks, that do not dwell by the sea, and that are not brave.

Of these sub-classes, however, not all are by the foregoing statements permitted to exist. Since *All Greeks are brave*, there can be no not-brave Greeks. Hence sub-classes 2 and 4 of the above list disappear from the world of possibility. Since *All Greeks are dwellers by the sea*, sub-class 3 vanishes. There remain as possible sub-classes of mankind, the first, fifth, sixth, seventh, and eighth sub-classes. The relations expressed in these sub-classes could be stated at length. Some of them are merely left possible, one is required by our original statement. For, as we see, the sub-class 1 must exist, since we have as required classes, *Greeks that are brave*, and *Greeks that dwell by the sea*. We have, therefore, the statement true, that *Some dwellers by the sea are brave*. The original statements tell us nothing of the fifth sub-class of our above list, which therefore remains merely abstractly possible. Therefore, we cannot tell whether there are any brave people that do not dwell by the sea, since the original statements give us no information about that matter. The whole meaning of our two original statements could thus be summed up:

(a) They *require* that the first sub-class of our list should be among possible classes. (b) They *forbid* that the second, third, and fourth classes of the list should be possible classes. (c) They *leave us uninformed* about the other sub-classes of our list, and these sub-classes may or may not be found possible. That is, again, they tell us that *if there exist in the actual world* Greeks, brave people and dwellers by the sea, then there *will exist* brave people dwelling by the sea, and these people will be Greeks; but there will in no case exist any Greeks dwelling by the sea and not brave, nor any Greeks that are brave, but that do not dwell by the sea, nor any Greeks that are neither brave nor dwellers by

the sea. But of other possible classes of mankind these statements tell us absolutely nothing. Thus, then, we have exhaustively stated the whole meaning of our two original statements.

The principle of our procedure has been as follows. Our two statements involved three terms, and told us something of the relations of these terms. To find the whole meaning of these statements, we merely have to find just what relations among their three terms the two statements involve. Now, if we knew nothing of the relations of the three classes, we could divide the whole world about which we are talking, as we divided any term in § 14, into eight sub-classes, formed by combining the three classes involved in our two statements. But since the three classes have some definite relations, therefore, as we saw in § 14, some of the eight sub-classes will disappear from the list, and there will remain only a part of the eight sub-classes. But to know what ones of the eight sub-classes are by our two statements caused to disappear, enables us to see some of the true relations of the three classes; just as seeing what part of the diagram was shaded out in column second of the table of class-relations in § 13 enabled us to see the true relations of the two classes x and y . Furthermore, our original statements require that some of the sub-classes should remain true classes, as real as the original classes *Greek*, *brave*, etc., themselves. Thus we learn yet more of the relations of the original classes, and thus we develop the whole meaning of our statements by seeing what sub-classes they together *require*, *forbid* and *permit* to remain as true classes.

This method is perfectly general. We could apply it to any number of statements. We can state it in a general rule, adding that for convenience sake we do well always to use letters as symbols for our classes:

RULE: *To find the combined meaning of several statements, first develop the full meaning of each statement separately, as in § 16. Then make of all the world, or of the part of the world concerning which we are speaking, an exhaustive division, according to all the terms used in the several statements. (Use for this purpose the table in § 14.) Then observe that if the name of any sub-class includes a combination of terms that is by any one of the statements forbidden, this whole sub-class must vanish. Discover in this way what sub-classes of our exhaustive division are*

caused to vanish. In like fashion, see whether our statements require that any of the sub-classes of the exhaustive division should remain as possible classes; and also whether there are any of the sub-classes concerning which no one of the several statements tells us anything whatever. The complete list of the classes that are excluded, of the classes that are required, and of the classes that are left doubtful by our several statements together, will be a full account of the meaning of these statements. We can express this meaning at pleasure in new statements.

Just as before we observed, in case of a single statement involving two terms, what ones of the sub-classes xy , $\bar{x}\bar{y}$, $\bar{x}y$, $x\bar{y}$, were excluded by the statement, what were left standing, what were left doubtful, so now we are to observe, in case of several statements, what possible sub-classes of things are by these statements excluded, what left standing, and what are left doubtful. We divide the world into sub-classes, and consider in detail what the statements tell us about each sub-class.

Notice the following principles:

If any combination of the exhaustive division contains a combination of letters that by the terms of our statements is equal to zero, this combination itself is equal to zero. Thus; if $xy = 0$, then any combination $wxyz$ must be $= 0$, and also $xvyz$ must be $= 0$, since the order of the letters is indifferent.

If any combination of letters xy is by one of our original statements required to exist, so that $xy > 0$ (as is the case in the statement *all x is y*), then *some one at least* of those combinations in the exhaustive division that contain xy must be > 0 . Thus of the combinations xyz xyz , one must in this case be *required*. In all other cases the combinations of the exhaustive division are left untouched, *i. e.*, are merely permitted.

Thus the problem that we set for ourselves at the outset of this book is solved, in so far as we intend to pursue the subject here. We have reduced statements to typical forms, we have examined each typical form to see what term-relations it involves, and then we have shown how by combining various statements we can discover what class-relations they together involve. Other problems regarding the meaning of statements there indeed are, especially in case of more complicated statements. But into these we cannot enter. Let the student look for them and for their solution

in Boole's *Laws of Thought*, in Prof. Jevons' *Principles of Science and Studies in Deductive Logic*, in Mr. Venn's *Symbolic Logic*, and in other similar modern books. Here has been attempted a discussion only of the *whole meaning of statements referred to their own subject- and predicate-terms*.

§ 18. APPLICATION TO LANGUAGES.

It remains to apply our results once more to actual language. We began with every-day speech. Analyzing it we were led to reduce it to certain typical forms. Analyzing these we discussed their meaning, and expressed this meaning symbolically. We have now to complete the circle by showing how our symbolic language may be applied to the forms of actual speech, and to the inferences of every-day discourse.

a. SUPPRESSION OF STATEMENTS.

If it is difficult to understand the full meaning of an ordinary sentence in speech, much more is it difficult to pierce through the many disguises that in actual speech hide the true nature of many sorts of inference. A statement can be inferred, we have seen from other statements, when it is implied in them as a part of their meaning. But often a statement is said in actual speech to be inferrible from some other statements when it is not contained in them alone as a part of their meaning, but is contained in them and in other not expressed but understood statements as a part of the meaning of all of these together. Inference from statements, some of which are expressed while some are suppressed, but are to be understood, is therefore a very common phenomenon. *Socrates is mortal, being a man*, is the short expression in speech for an inference from *all men are mortal*, and *Socrates is a man*. These two statements together imply that *Socrates is mortal*. If the student does not at once see that they do imply this last statement, let him apply our previous rule for discovering the meaning of two statements, and he will quickly see that they do imply what we have inferred.

Logical analysis of collections of statements must therefore be prepared to supply, in case of all inferences that occur in speech, all the suppressed statements that are understood as a part or ground of the inference. These suppressions in speech are of

three classes: (1) One or more of the statements from which we are to infer a new statement are suppressed, the others being given in full; (2) All the statements from which we infer are as separate statements suppressed, and we are left with the inferred statement and only a hint of the statements from which it came, or with a phrase or clause to represent one of these statements; (3) The inferred statement is suppressed, and we are left with several statements, whose combined meaning is so obvious as to be understood at once. An example of (1) was given above. An example of (2) is: *It will certainly rain from the black sky.* The suppressed statements are: *From black skies rain is certain. That sky is black.* Examples of (3) are common for rhetorical effect. Innuendo, for example, is a form of inference in which the inferred statement is left to be understood. Another example would be: *All great cities are full of misery, and London is the greatest of cities.* A most important part of the analysis of speech is the supplying of suppressed statements. If you understand a chain of reasoning you can always do this, but not otherwise.

b. DISJUNCTIVE SENTENCES.

A disjunctive sentence may enter into a chain of inferences, and needs special attention. We have before seen that disjunctive sentences are composite. We have put off until this point the analysis of their full meaning. A disjunctive sentence says that any class as x is made up of several other classes, which may or may not exclude one another, but which together make up the class x . Thus the sentences: *Heavenly bodies are self-luminous or not self-luminous; rulers are emperors, or kings, or dukes,* are both disjunctive sentences. If, in disjunctive sentences the classes connected by *or* are not exclusive, we can always restate the sentence so that the classes connected by *or* shall be exclusive. Thus, in place of the second sentence above, we might write the sentence: Rulers are (1) emperors, or (2) they are kings that are not also emperors, or (3) they are dukes that are neither kings nor emperors. Here the classes are exclusive, and together equal to the class *Rulers*. If, now, a disjunctive sentence is written so that its classes are mutually exclusive, then its interpretation is very easy. Suppose we have the statement x is either y or z , where y and z are classes that are mutually exclusive, so

that we have $yz=0$, and $\bar{y}z>0$, $\bar{y}\bar{z}>0$. Then from the exhaustive division of x expressed in the equation $x=xyz+\bar{x}yz+\bar{x}\bar{y}z+\bar{x}\bar{y}\bar{z}$, there disappears the class xyz , while the class $\bar{x}yz$ remains untouched, and while the classes of the division $\bar{x}\bar{y}z$ and $\bar{x}\bar{y}\bar{z}$ are required as possible classes. The meaning of the disjunctive sentence is now clear. We might state it in two simple statements, thus: An x that is not y is z , and an x that is not z is y . To find, then, the meaning of a disjunctive statement, we need to restate it in such a form as to make the classes connected by *or* exclusive. These classes will, then, together, furnish the exhaustive division of the subject-term, and the meaning of the statement will forthwith appear.

c. FALLACIES OF INFERENCE.

Into the large subject of fallacies these lessons do not seek to enter. Suffice it to say, that by fallacy is meant an apparent inference that is not actually an inference. To detect a fallacy is merely to see that two or more statements do not separately or jointly mean what they have been supposed to mean. Detection of fallacy depends, therefore, upon correct logical analysis of the meaning of language, and fallacy expressed in language can deceive nobody that knows what the language means. In examining any argument or expressed inference, you have, then, merely to ask yourself this question: Do the statements here made or understood actually include, as a part of their meaning, this that is to be inferred from them? If they do, the statements may be false, but the inference is good. If they do not, the inference is bad, even though all the statements, and all that is inferred from them, should be true.

CONCLUSION.

These general methods of analyzing the meaning of language, are too cumbersome for every-day use, but the study of them is none the less valuable. For these methods express reflection upon the meaning of speech, reflection that, if persevered in, must end in making our own speech clearer, and our reading and listening more profitable. The mere consciousness that speech is always and everywhere concerned with the *relations of classes* is a most useful acquirement. As a fact, we have seen that some one of the relations of § 13 is always affirmed to exist between the sub-

ject and predicate terms used in our language, whenever we make any declaration, no matter what we are talking about, or what we say. And so the habit of cross-questioning ourselves, of continually asking ourselves: *What class-relations do we mean to affirm in this that we have said?* is an invaluable habit. By it our thoughts are compelled to become sharp and definite, instead of remaining confused and misty. And clear writing is the first-born daughter of clear thinking. If these lessons prove in the least helpful to any student that wishes to form this habit, they will have done very good work.

EXERCISES ON CHAPTER V.

[The object here is not to give numerous examples, for such are elsewhere to be found, but merely to enforce a few of our principles by simple illustrations.]

1. Develop the whole meaning of each of the statements that were contained in the exercises of chapter III.

NOTE: To save writing, symbolize the terms by letters, express the whole meaning in these, and be prepared to translate the results back into common language.

2. From *All tyrants are blameworthy*, does it follow that no not-tyrants are blameworthy?

3. Can we infer any new statement from the two statements, *No x is z*, *No y is z*?

4. Is this a correct inference: If you fall into the water you will be drowned. But you will not fall into the water. Therefore you will not be drowned?

5. $xy=0$, $x\bar{y}z=0$, $\bar{x}yzu>0$, $\bar{z}u=0$, $zu>0$, $\bar{y}z>0$. What is the whole meaning of these statements taken together?

NOTE: Use the table in section 14, writing x , y , z , and u for p , q , r , s , and t .

6. Tell concerning each of the following statements whether it is consistent with, or equivalent to, or deducible from, or contradictory to the statement, *All just men are happy*:

All unjust men are unhappy.

Some happy men are unjust.

Some unjust men are happy.

No unhappy men are just.

No unjust men are happy.
All happy men are unjust.
All unhappy men are just.
Some just men are not happy.
Some happy men are just.

NOTE: The circle-notation, as a means of expressing the meaning of these statements, will possibly be the best for the present purpose.





3 2044 025 696 212